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OPTIMISATION OF WORK OF SCREW IMPELLERS IN A DIFFUSER BY MEANS OF THE EVALUATION OF MIXING EFFECTIVENESS*)

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Abstract. The measurements of instantaneous velocity distributions in a liquid of high viscosity while its mixing in a mixer were carried out with the application of screw impellers in diffusers. The measurements were performed using a laser anemometer (BSA, Dantec) and the LDA method. On the basis of the experiments and a 3D dimensionless mathematical model of the mixer with a screw impeller, proposed by the Author, the determination of the experimental and theoretical circulation times τ_c, homogenization times τ_m and the value of secondary circulation V_s was performed in the mixer. The experimental and theoretical dimensionless modules of circulation times K_c and homogenization times K_m were determined. Furthermore, the value of mixing power P and actual amount of energy transferred to the mixer – impeller system were determined. The Author proposed his own definition of effectivity $e_m=P\cdot\tau_m$. The effectivity defined in this way was used to optimize the mixer – impeller system in the investigated scope of geometry.

Key words: Mixing process, screw impeller, modelling, draught tube, high viscous liquid, vessel with a stirrer;

1. INTRODUCTION

One of the requirements of mixing is a maximal shortening of the time of the process which demands little energy. In the case of screw and ribbon impellers the time of homogenisation generally depends on the circulation time. It must be added that both parameters depend on secondary circulation in a mixer.

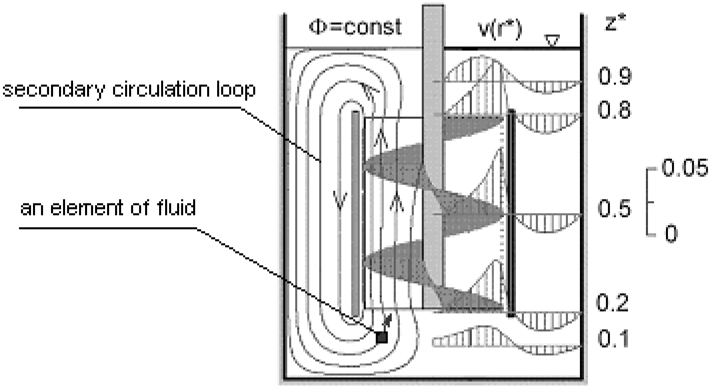

Fig. 1. Secondary circulation in a mixer

Secondary circulation V_s in a mixer may be understood as the volumetric intensity of the liquid flow in the r-z plane (radial – axial) which is strictly connected with pumping capacity of impellers. It is assumed that secondary circulation in the mixer is two times higher than its pumping capacity [1]. In practice, secondary circulation is considered to be the volumetric flux of the liquid flowing through a ring the inner diameter of which is indicated by the centre of the circulation loop. The external diameter of the ring is marked by the wall of the mixer. The ring is located in the plane normal to the axis of the mixer at the height of the circulation

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loop centre. In the case of screw impellers the centre of circulation loop overlaps with the diffuser wall due to the fact that it constitutes a stationary element the mixed liquid circulates about. The circulation time τ_c may be understood as the time which is necessary for an element of fluid Fig. 1 (moving with defined mean velocity $\overline{v}$) to achieve a distance equal to one full circulation loop in the mixer. The mean circulation time τ_c may be calculated (1) dividing the mixer volume $V_{zb}[m^3]$ by secondary circulation $V_s[m^3/s]$.

$$\tau_c = \frac{V_{zb}}{V_s} \tag{1}$$

The time of homogenisation $\tau_m[s]$ may be regarded as the time needed to obtain a defined degree of homogenization in the mixer being appropriate from the technological point of view. In the papers [2,3] it is stated that the homogenization time τ_m is strictly connected with the circulation time τ_c both in the turbulent and laminar flow by means of a simple functional equation. Furthermore, the homogenization time may be treated as the multiplication of the circulation time. The dependence may be defined using the following general equation:

$$\tau_m = \beta \cdot \tau_c \tag{2}$$

The functional dependence, which may be found in the papers [2,3] and define the relation between the homogenisation and circulation time in the laminar flow, shows a simple proportionality between the two parameters. The value $\beta[-]$ defines a number of circulation loops to be performed by an element of fluid so that the liquid obtains an adequately high degree of mixing. Thus, assuming, similarly as for the laminar flow, a simple relation between the values τ_m and τ_c on the basis of the available literature data, the author established one's own value of the proportionality coefficient β:

$$\beta = \frac{\tau_m}{\tau_c} = 3\frac{1}{3} \tag{3}$$

The way of how it had been obtained was defined in the study [3]. Nevertheless, such a value of the proportionality coefficient $\beta \cong 3.33$ should be regarded as an indicatory one. As evidenced in the paper [1], the value of the proportionality coefficient β in the turbulent flow oscillates about $4 \div 5$. The lower value of the coefficient obtained for the laminar flow is not equivalent to, generally being shorter, the times of homogenization when compared to the turbulent flow. Therefore, the values of the coefficients β should not be compared (the times of circulation attain many times smaller values in the turbulent flow). Additionally, the time of homogenization τ_m and time of circulation τ_c may be defined using the appropriate dimensionless modules of the homogenization time $K_m[-]$ and circulation time $K_c[-]$.

$$K_c = \tau_c \cdot N \tag{4}$$

$$K_m = \tau_m \cdot N \tag{5}$$

The dimensionless module K_c defines such a number of mixer's rotations with which an element of liquid achieves a distance of one circulation loop. The dimensionless time of homogenization, on the other hand, defines a number of mixer's rotations being necessary to obtain a defined degree of homogenization [3]. An increase in secondary circulation brings about a decrease in circulation time which, in turn, contributes to a decrease in mixing time. Nevertheless, the drive of an impeller requires a supply of defined energy depending on the structure of the impeller, its geometrical parameters, rotation frequency and the physical-chemical properties of mixed liquid. Screw and ribbon impellers as a class of close-clearance impellers, generally consume a lot of energy. Furthermore, due to their purpose (for mixing of high viscous liquids) an increase in viscosity of liquid is connected with a linearly proportional increase in the mixing power. Moreover, for the laminar regime of mixing, an increase in rotations of the impellers contributes to an increase in the mixing power to the second power of rotations $P \sim N^2$, and an increase in diameter increases the mixing power in proportion of $P \sim d^3$. The aforementioned factors contribute to the fact that there is no possibility of optional shortening of the mixing time by an optional manipulation of impeller's geometry or regulation of rotations. This is due to the fact that a relevant parameter, which has to be considered from the point of view of the homogenization time, is the mixing power. The mutual relationships between the mixing power and homogenization time prove the operating effectiveness of a given impeller. Hence, the smaller the power needed to obtain a predefined technological effect or the shorter homogenization time for given impellers with their unchanged power intake, the more effective work of a given impeller. On the basis of the aforementioned factors one may derive an energetic criterion of the mixing effectivity e_m which is the product of the homogenization time τ_m and mixing power P. As a result, it defines the so-called energy of mixing, in other words an amount of energy that has to be provided to a defined system to obtain an appropriate, from the technological point of view, degree of homogenization.

In the laminar flow the dimensionless homogenization times K_m and circulation times K_c are constant and do not depend on the Reynolds number [-]. Thus, for this range of the Reynolds number the following dependence is valid:

$$\tau_m = \frac{K_m}{N} \tag{6}$$

In addition, knowing that in the laminar flow the dependence $Ne \cdot Re = A$ is valid, the mixing power may be calculated from the following equation:

$$P = A \cdot N^2 \cdot d^3 \cdot \eta \tag{7}$$

applying the classical definitions of the Reynolds number Re and power Ne described in [1,2]. Where: d - the diameter of an impeller [m], η - viscosity [Pa·s].

Hence, the proposed energetic criterion e_m acquires the following form:

$$e_m = P \cdot \tau_m = A \cdot K_m \cdot N \cdot d^3 \cdot \eta \tag{8}$$

The effectiveness e_m[J] defined in his way may be used to optimize the system: impeller – mixer. The optimization in the case of the homogenization time defines the selection of system's geometry in such a way that with the unchanged power intake one could obtain the best intensity of mixing, i.e. the shortest mixing time. As evidenced in Eq. (8) the lower the value of effectivity e_m, the more effective operation of a given impeller.

2. EXPERIMENTAL

The investigations concerning the measurements of the velocity components in circumferential and axial directions, in the region of an impeller for screw impellers operating in a diffuser, in the laminar flow, were carried out. The main element of the measurement system was: the BSA doppler laser anemometer (Burst Spectrum Analyser) (Dantec). The examinations connected with mixing power were performed using a measuring apparatus MR-D1 (IKA Labortechnik Staufen) (measurement error -1.5%, the reproducibility of the results +/- 0.5%). A detailed description of the investigation and research stand is attached to the paper [2]. The investigations were carried out for five screw impellers differing in terms of the diameter and pitch of a ribbon equipped with the diffusers being dependent on the diameters of the impellers. In each case the screw impeller was centrally located inside the mixer and mounted at half of the height of the tank. The parameters of the impellers under scrutiny are summarized in Table 1. The viscosity was measured using the Rheotest 2 apparatus and Hoeppler viscosimeter.

Table 1. Parameters of screw impellers

Impeller	Diameter d [m]	Height h [m]	A number of coils i [-]	Pitch of screw p [m]
S1	0,084	0,170	2	0,085
S2	0,135	0,170	1	0,170
S3	0,135	0,170	2	0,085
S4	0,135	0,170	3	0,056
S5	0,164	0,170	2	0,085

A flat-bottomed glass tank of diameter D=0,292 [m] was filled with liquid to the height equal to the diameter of the tank H/D=1. The liquid investigated was the solution of Optima potato syrup of viscosity $0,43\leq\eta\leq8,06$[Pa·s]. The density of the syrup ρ was 1350±5 [kg/m^3]. A change in the hydrodynamics of the system, and , simultaneously, a change in the Reynolds number Re, was obtained by means of the change of rotations N of the impeller for the established viscosity η, or by means of a change in viscosity η for the driving system operating with a constant rotation frequency ω [1/s].

3. RESULTS AND DISCUSSION

In accordance with the aim of the study the evaluation of the operation effectiveness of the investigated screw impellers was performed from the homogenisation time point of view and based on a 3D solution of the TSN (three-dimensional numeric simulation) dimensionless model presented in the study and the experimental investigations.

In Table 2 one may find the dimensionless numbers K_s, K_c, K_m, circulation time τ_c and homogenization time τ_m for the investigated screw impellers, obtained on the basis of the experimental mean axial velocity distributions in a radial direction presented in the papers [2,3]. The way of calculation of those velocities has been described. The prerequisite to obtain their numeric values was the definition of secondary circulation in the mixer V_s[m^3/s]. The calculation of V_s slightly differs for screw impellers or ribbon impellers but in both cases the value V_s is given in the form of the dimensionless number of secondary circulation K_s[-], defined by the equation:

$$K_s = \frac{V_s}{N \cdot d^3} \tag{9}$$

In the case of the screw dryer operating in the diffuser the secondary circulation in the mixer is equal to the volumetric intensity of the liquid flow in the diffuser or in the region of a ring (between the wall of the impeller and diffuser) in an axial direction which may be observed in Fig.1. Hence, the experimentally determined mean axial velocities distribution (using the LDA doppler anemometer) along the radius of the mixer at the height of z*=0,5 (in the region of the diffuser and ring) was integrated (the equations 10, 11) in order to calculate the volumetric intensity of the liquid flow in those regions.

$$V_{s\,dyf.} = 2 \cdot \pi \cdot \int_{\frac{d_p}{2}}^{\frac{d_w}{2}} \left(r \cdot \bar{u}_v(r)\right) dr \tag{10}$$

$$V_{s\,ciecz} = 2 \cdot \pi \cdot \int_{\frac{d_z}{2}}^{\frac{D}{2}} \left(r \cdot \bar{u}_v(r)\right) dr \tag{11}$$

where $V_{s\,dyf}$ – the volumetric intensity of the liquid flow in the area of the diffuser, $V_{s\,ciecz}$ – the volumetric flow of the liquid in the area of the liquid [m^3/s], D – the inner diameter of the mixer [m], $\overline{u}_v$ - the mean actual axial velocity [m/s], r – the actual radius in the tank [m], d_w – the inner diameter of the diffuser [m], d_z – the external diameter of the diffuser [m].

In Table II one can find the experimental results of the dimensionless numbers K_s, K_c, K_m as well as the circulation times τ_c and homogenization times τ_m obtained for the screw impellers under scrutiny in the selected areas of the mixer (the area of the diffuser and liquid). To calculate the homogenization time τ_m the value of the coefficient $\beta=3\frac{1}{3}$ defined by the Eq. (3) was used. Due to minor measurement errors, the experimental values of secondary circulation for the region of the diffuser and liquid only slightly differed, thus in further sections of the chapter the mean values of secondary circulation obtained from both regions were applied to evaluate the effectivity of screw impellers operation. Those values in combination with the model values obtained in an analogous way, but on the basis of the solution of the presented 3D model [2], are summarized in Table III.

According to the expectations, the shortest times of homogenization were obtained for the impeller of the greatest diameter. This fact was connected with the highest value of secondary circulation V_s for this impeller (Table II). What is more, the shortest times of homogenization for the pitch of the screw p/d=1.26 were obtained for the impellers of constant diameter d=0.135 due to the fact that it is this type of an impeller that displayed the highest secondary circulation, which is in agreements with the experimental results [4]. It must be underlined that Table II does not encompass the energy consumption of the individual impellers.

Table II. The experimental dimensionless numbers K_s , K_c, K_m and circulation times τ_c and homogenization times τ_m for screw impellers for N=1.04 [s^{-1}] and liquid viscosity η=4 [Pa·s].

Diameter of an impeller d [m]	Number of coils i [-]	p/d [-]	Liquid area	$V_{s\,ciecz.}$ $\left[\frac{m^3}{s}\right]$	K_s [-]	τ_c [s]	τ_m [s]	K_c [-]	K_m [-]
			Diffuser area	$V_{s\,dyf.}$ $\left[\frac{m^3}{s}\right]$	K_s [-]	τ_c [s]	τ_m [s]	K_c [-]	K_m [-]
0,135	1	1,26	Liquid area	0,00069	0,2626	28,5	95,0	30,3	100,8
			Diffuser area	0,00065	0,2480	30,2	100,7	32,0	106,7
0,135	2	0,63	Liquid area	0,00048	0,1843	40,7	135,5	43,1	143,6
			Diffuser area	0,00045	0,1715	43,7	145,6	43,3	154,3
0,135	3	0,42	Liquid area	0,00034	0,1309	57,3	190,8	60,7	202,2
			Diffuser area	0,00030	0,1157	64,8	215,8	68,7	228,8
0,164	2	0,52	Liquid area	0,00075	0,1573	26,1	87,0	27,7	92,2
			Diffuser area	0,00080	0,1672	24,6	81,8	26,0	86,7

0,084	2	1,01	Liquid area	$6{,}9\cdot10^{-5}$	0,1101	282,6	941,1	299,6	997,6
			Diffuser area	$8{,}3\cdot10^{-5}$	0,1320	235,9	785,5	250,0	832,6

Table III. Experimental and model mean values of homogenization times $\bar{\tau}_m$ and dimensionless numbers $\overline{K_m}$ of screw impellers for N=1.04 [s^{-1}] and liquid viscosity η=4 [Pa·s]

d/D	Diameter of an impeller d [m]	Number of coils i [-]	p/d [-]	Experimental $\overline{K}_m$ [-]	Model K_m [-]	Experimental $\bar{\tau}_m$ [s]	Model τ_m [s]
0,462	0,135	1	1,26	103,8	110,3	97,8	104,1
0,462	0,135	2	0,63	149,0	184,7	140,6	174,3
0,462	0,135	3	0,42	215,5	262,3	203,3	226,2
0,562	0,164	2	0,52	89,4	68,4	84,4	64,5
0,289	0,084	2	1,01	915,1	989,3	863,3	933,3

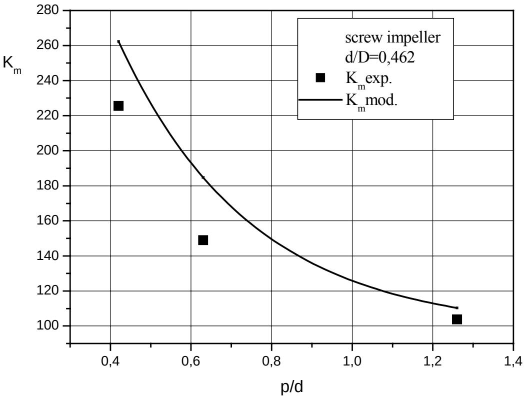

Figure 2. Comparison of dimensionless homogenisation times K_m in a pitch invariant function p/d for a screw impeller.

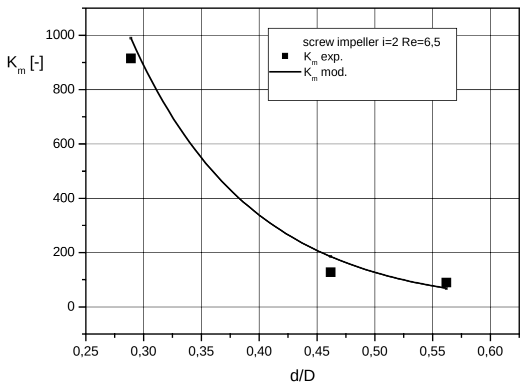

Figure 3. Comparison of mean dimensionless homogenization time $\overline{K_m}$ in a diameter invariant function d/D for a screw impeller.

In Figure 2 the experimental mean dimensionless values of the homogenisation times $\overline{K_m}$ and model values K_m (obtained on the basis of the Author's own model (3D) described in [2]) in a diameter pitch function p/d for the screw impeller of diameter invariant d/D=0.462 are compared. Analogously, in Figure 3 the mean experimental and model values of the homogenization times K_m in a diameter invariant function d/D for the impeller of coil number i=2 are compared. The mean differences between the experimental and model values K_m are ~8% and the value of K_m decreases with an increase in pitch and diameter of a screw which is in agreement with the conclusions in the literature on the subject [1-14].

As it has been previously mentioned, the evaluation of screw impeller operating effectivity regarding the homogenization times requires the mixing power to be considered. In Table IV the experimental and model values of the mixing times τ_m,, mixing power P and the values of the introduced energetic mixing criterion $e_m=(\tau_m \cdot P)$ [J] are summarized. The criterion allows to find an optimal system regarding both the power and time of homogenization. The analysis is devoted to finding the energetic extremum of the minimum of the function.

$$e_m = \left(\tau_m \cdot P\right)_{\substack{d/D \\ p/d}} \rightarrow \min . \qquad (12)$$

Table IV. Mean experimental and model values of homogenization times $\tau_{m,}$ mixing power and an energetic criterion e_m for the investigated screw impellers, for N=1.04 [s^{-1}] and liquid viscosity η=4 [Pa·s].

d/D	Diameter of an impeller d [m]	Number of coils i [-]	p/d [-]	Re [-]	Exp $\bar{\tau}_m$ [s]	Model. τ_m [s]	P [W]	Experimental $e_m = \left(\bar{\tau}_m \cdot P\right)$ [J]	Model $e_m = \left(\tau_m \cdot P\right)$ [J]
0,462	0,135	1	1,26	6,4	97,9	104,1	2,38	232,3	247,1
0,462	0,135	2	0,63	6,4	140,6	174,3	2,71	381,6	473,1
0,462	0,135	3	0,42	6,4	203,3	226,2	4,26	865,6	963,1
0,562	0,164	2	0,52	9,4	84,4	64,5	5,57	470,1	359,6
0,289	0,084	2	1,01	2,5	863,3	933,3	0,90	779,2	842,4

From the analysis of Table IV it may be concluded that an optimal screw impeller regarding the effectivity criterion was the impeller of diameter invariant d/D=0,462 and pitch invariant being equal p/d=1,26 (the frame in bold). The key element in this case was too high value of secondary circulation V_s for this screw (Table III). The pitch of the impeller is approximate to the value of the optimal pitch ratio described in the subject literature [4] as p/d$\cong$1,3. The above considerations imply that the screw impellers of too small diameters should not be used due to the fact that their small size contributes to the formation of the incomparably small secondary circulation in the mixer.

4. CONCLUSIONS

The analysis allows to draw the following conclusions:

1. On the basis of a simple proportionality of the circulation and homogenization times and using the model solutions the optimization of screw impellers regarding the homogenization time may be carried out.
2. In the case of screw impellers the homogenization time increases with a decrease in the pitch of a ribbon and decrease in impeller's diameter (within the investigated changes of the geometrical parameters of the impeller).
3. In the case of screw impellers the mixing power increases with a decrease in the clearance between the edge of the impeller and internal wall of the diffuser and with a decrease in the ribbon pitch of the impeller.

4. The optimal parameters regarding the homogenization times may be attributed to the screw impeller, the diameter and screw pitch ratio of which were only slightly different from $d/D \cong 0{,}5$ and $p/d \cong 1{,}26$.
5. The possibility of using the model of the mixer operating in the laminar flow in 3D space, proposed in the study [2] for full optimization of screw impellers, regarding homogenization time has been confirmed.
6. It has been stated that, in practice, the screw impellers of too small diameters should not be used due to the fact that their small size contributes to the formation of incomparably small secondary circulation in the mixer.

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