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DETERMINATION OF TURBULENT DIFFUSION COEFFICIENT IN AN AGITATOR FOR SPECIAL TYPE OF IMPELLER

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Abstract. In the study the analysis of the distribution of basic parameters in turbulent momentum and mass transport is presented, in particular, the analysis of the distribution of viscous and turbulent stress, the distribution of turbulent viscosity and the turbulent diffusion coefficient in the mixer with a self-aspirating impeller were performed. The calculations of the aforementioned parameters were carried out on the basis of the measurements of mean and fluctuating velocities in a vertical cross-section of a mixing vessel with one-phase liquid system.

Key words: Turbulent mixing, turbulent viscosity, turbulent diffusion coefficient

1. INTRODUCTION

While mixing of two phase gas-liquid or liquid-liquid systems the basic parameter one attempts to determine is mass flux j_A [kg/(m^2·s)]. This is due to the fact that on this basis one may determine the mass of component A transporting from one phase to another one. Simultaneously, the quantity of j_A strongly depends on the structure of the continuous phase as well as the structure of the flow. In the model of homogeneous flow [1], which does not consider the presence of the interface surface and treats the whole mixture as the continuous with a constant density ρ, the momentum and mass balance equation for the component A takes form of two equations (1) and (2):

$$\rho \frac{D\overline{\mathbf{w}}}{Dt} = -\nabla \overline{p} - \nabla \overline{\mathbf{T}} - \nabla \overline{\mathbf{T}^t} \tag{1}$$

$$\frac{D\overline{c_A}}{Dt} = -\nabla \overline{j_A} - \nabla \overline{j_A^t} \tag{2}$$

In the equation (2), which defines the rate of mass transfer, the mean unit mass flux j_A describes the molecular transport which may be defined by the Fick's equation. To exemplify, for y direction it is in the following form:

$$\overline{j_{Ay}} = -D_{Ay} \cdot \frac{\partial \overline{c_A}}{\partial y} \tag{3}$$

On the other hand, the mean unit mass flux j_A is connected with the fluctuation of the velocity and concentration field which, after considering the Boussinesque hypothesis, enables to obtain the following equation (4)

$$\overline{j^t_{Ay}} = \overline{v_y{}' \cdot c_A{}'} = -D^t_{Ay} \cdot \frac{\partial \overline{c_A}}{\partial y} \tag{4}$$

Thus, one may conclude that the total mass flux may be defined using the effective diffusion coefficient D^{ef}_{Ay}:

$$j_{Ay} = \overline{j_{Ay}} + \overline{j^t_{Ay}} = -(D_{Ay} + D^t_{Ay}) \cdot \frac{\partial \overline{c_A}}{\partial y} = D^{ef}_{Ay} \cdot \frac{\partial \overline{c_A}}{\partial y} \tag{5}$$

the analogous calculations may be performed in the case of the momentum balance equation (1) which results in the following equation for the y direction, for example

$$\tau^{ef}_{yx} = \tau_{yx} + \tau^t_{yx} = \mu \cdot \frac{\partial \overline{v_x}}{\partial y} + \rho \cdot \overline{v'_x \cdot v'_y} = -(\mu_{yx} + \mu^t_{yx}) \cdot \frac{\partial \overline{v_x}}{\partial y} = -\mu^{ef}_{yx} \cdot \frac{\partial \overline{v_x}}{\partial y} \tag{6}$$

Many experimental inwestigations indicate [2,3] that the equivalent mechanism of all transport phenomena in the scope of turbulent motion exists, which means that the turbulent coefficients of the momentum and mass transfer do not differ considerably between themselves. This situation is defined using the turbulent Schmidt number Sc

$$Sc^t = \frac{\nu^t}{D^t_A} \tag{8}$$

Often, it is assumed that $Sc^t \cong 1$. In the case when $Sc^t = const.$ defining one turbulent coefficient allows to determine another one which, in consequence, leads to the equation (9)

$$D^t_A = \frac{\mu^t}{\rho} \tag{9}$$

In this way the turbulent difussion coefficient D^t_A may be defined on the basis of the turbulent field structure in one of the phases.

In the case of two-phase gas-liquid systems the determination of the turbulence structure of liquid phase is impeded due to the presence of interface surface. The interface surface decreases turbulence of liquid phase and, what is also relevant, it contributes to problems in measurements. In particular, it concerns the case in which the measurements of instantaneous velocities are performed using the LDA laser system due to the fact that the system cannot recognize in which phase the measurement point is to be found. To omit those inconveniences, in the cases in which an amount of gas supplied to the agitator is relatively small, it is assumed that a change in turbulence of liquid phase brought about by the presence of interface surface is also small. This way, with a certain simplification, a two-phase system may be treated as one–phase system from the hydrodynamic point of view.

Self-aspirating impellers which aspirate gas from the free surface of liquid into the inside of the agitator are characterized by rather weak or moderate volumetric flow rate of gas. Considering self-aspirating impellers, operating at the onset of self-aspiration, the amount of aspirated gas is particularly small and the assumption of similarity of turbulence structure in one- and two-phase system may be applied.

2. AIM OF THE STUDY

The aim of the study was to experimentally determine the parameters of turbulent flow in an agitator vessel with a self-aspirating impeller [4,5,6] and on the basis of this to calculate the viscous and turbulent stress as well as the distribution of the local turbulent viscosity and turbulent diffusion coefficient inside the agitator.

3. EXPERIMENTAL

The main element of an experimental apparatus was a glass, flat-bottomed, cylindrical tank of a diameter D=292mm with an axially mounted self-aspirating impeller of a diameter d=125 mm located at the height h=62 mm from the bottom. The agitator vesel with four standard baffles of width 1/10·D was filled with liquid to the height H=D. In Fig. 1 the basic dimensions of the agitator and impeller are presented.

The measurements of instantenous velocities were performed using a laser Doppler anemometer (DANTEC) equipped with 100mW laser (the wavelength =514.7 nm) and a signal processor BSA T58N10. The measurements of instanteous velocities were carried out in the radial-axial plane in about 180 points. The frequency of sampling attained the values from 0.5 kHz closely to the surface of the liquid to 4kHz in the impeller zone.

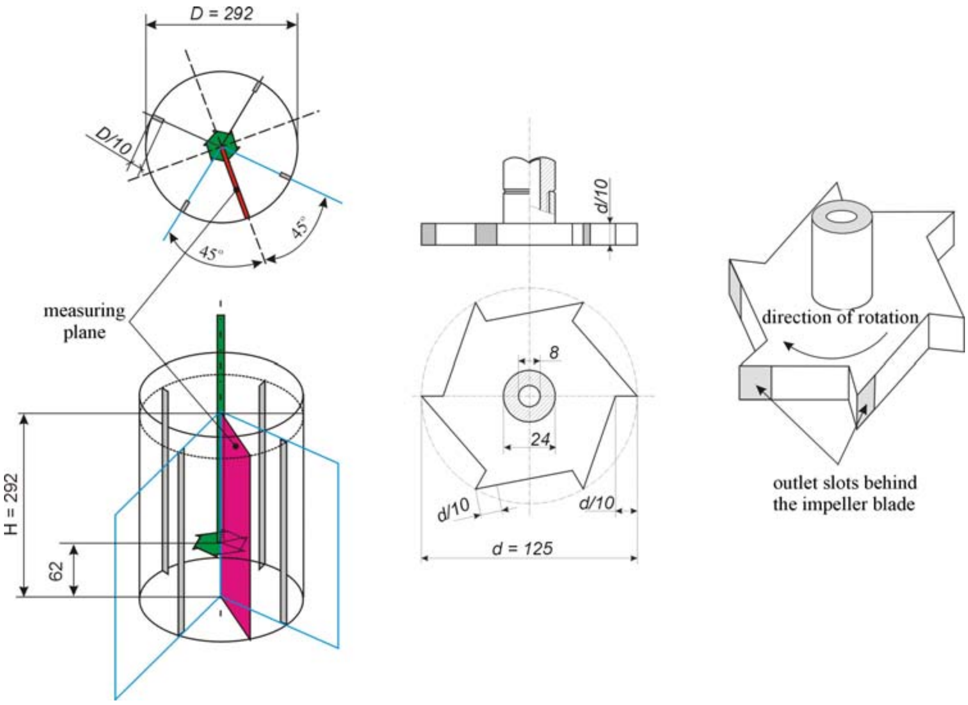

Fig. 1. Basic dimensions of an agitator vessel and impeller.

All measurements were carried out for water ($\mu=10^{-3}$ Pas, $\rho=1000$ kg/m^3), for the rotating frequency of the impeller N=365 min^{-1} which is equivalent to the Reynolds number Re=95050 and this is simultaneously the onset of self-aspiration for this type of the impeller. The measurements were done for the one-phase system, in other words with covered air slots in the wall of the agitator.

4. RESULTS

4.1 Local stress

In Fig. 2 the distributions of all velocity coordinates along the radius of the agitator and at its height are presented. It may be easily perceived that, excluding the agitator zone and its closest surrounding, the turbulence in the agitator may be regarded as izotropic and only individual mean velocity components considerably differ between themselves.

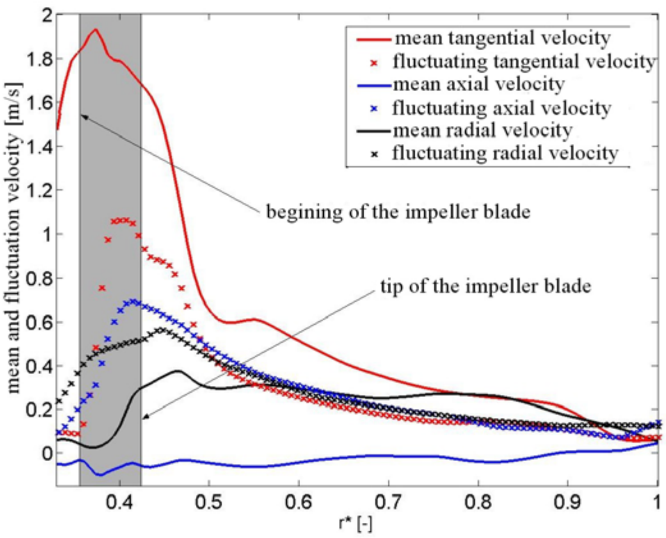

Fig. 2. Comparison of the distributions of mean and fluctuating velocities for individual velocity components at the height of impeller suspension

In accordance with the expectations, mean circumferential velocity is the highest and axial velocity is the smallest. The character of changes of mean and fluctuating velocities along the height and radius of the agitator is in agreement with the data presented in the other papers [6, 10, 11].

On the basis of the measurements of mean velocities at different points in the mixing vessel (Fig. 1) total viscous stress was defined. The stress is understood as the sum of the individual components of tensor T defined by the equation (9). The tensor defines the sum of shear and normal stresses in the individual directions of the coordinate system.

$$\overline{\mathbf{T}} = \mu \cdot \begin{bmatrix} \frac{\partial \overline{v_x}}{\partial x} & \frac{\partial \overline{v_y}}{\partial x} & \frac{\partial \overline{v_z}}{\partial x} \\ \frac{\partial \overline{v_x}}{\partial y} & \frac{\partial \overline{v_y}}{\partial y} & \frac{\partial \overline{v_z}}{\partial y} \\ \frac{\partial \overline{v_x}}{\partial z} & \frac{\partial \overline{v_y}}{\partial z} & \frac{\partial \overline{v_z}}{\partial z} \end{bmatrix} \tag{9}$$

The results of the calculations are demonstrated in Fig. 3. In the figure the results obtained at 180 measurement points in the plane r-z are presented which were, subsequently, interpolated to the whole measurement plane r-z. The interpolation was carried out using the method of biharmonic spline curves.

Fom the calculations one may infer that the greatest values of the viscous stress τ (as regards the absolute value) may be encountered at the bottom and on the upper edge of the impeller paddle. In this zone the values of viscous stress attain the level of ± 0.25 Pa. What is more, the change of the sign of stress may be observed which results from the presence of two vortices of the secondary circulation rotating in the opposite directions (the vortex above and under the impeller). Beyond the impeller zone the values of viscous stresses oscillate about $\tau = 9 \cdot 10^{-4}$ Pa, in other words being three orders smaller than those present in the impeller zone. The similar character of changes were obtained in the study [7].

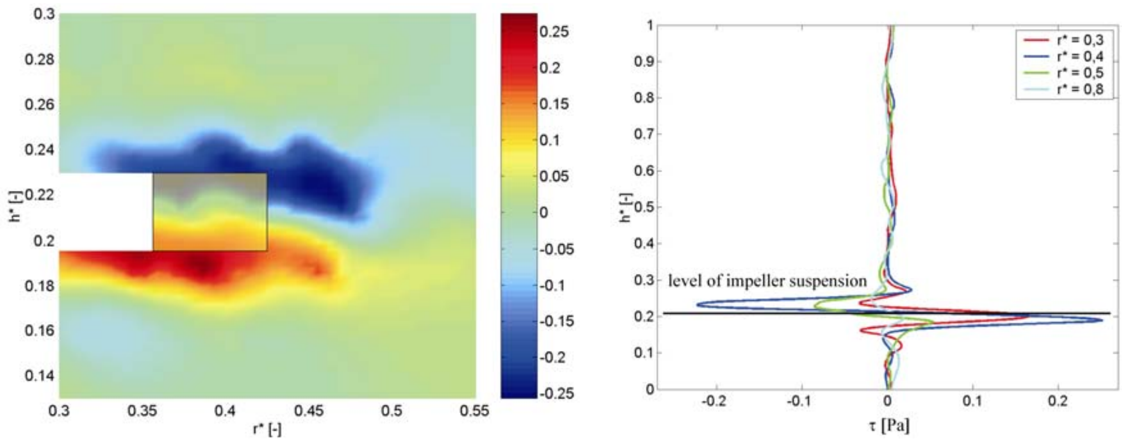

Fig. 3. Total viscous stress τ [Pa]

The value of the second tensor $\overline{\boldsymbol{T}^t}$ being present in the equation (1) is dependent on the character of turbulence, in other words on the fluctuating velocities. The tensor [8] for all directions of the coordinate system is defined by the following equation (10):

$$\overline{\mathbf{T^t}} = \rho \cdot \begin{bmatrix} \overline{v'^2_x} & \overline{v'_x \cdot v'_y} & \overline{v'_x \cdot v'_z} \\ \overline{v'_y \cdot v'_x} & \overline{v'^2_y} & \overline{v'_y \cdot v'_z} \\ \overline{v'_z \cdot v'_x} & \overline{v'_z \cdot v'_y} & \overline{v'^2_z} \end{bmatrix} \qquad (10)$$

The greatest changes were also expected within the region of the paddle of the agitator [9] and those predictions are confirned by the experimental data in Figure 4. It may be easily perceived that the greatest values of turbulent stress are present at the level of impeller suspension and on the external edge of the paddle they attain the value of 5000 Pa.

Moving away from this surface the value τ^t rapidly decreases and at a distance of about 10mm from the edge of the paddle it is 10 times smaller that its maximal value. In the remaining zone of the agitator one observes no significant changes of turbulent stress along the radius and height of the agitator, and the values of τ^t oscillate around $\tau^t \cong 70$ Pa.

In the study [11] maximal turbulent stresses of the value 3000 Pa in the zone of the paddle of the Rushton turbine of a diameter d=0.333m for the rotating frequency N=180 min^{-1} were observed. What is more, it must be mentioned that this value rapidly decreases with a distance from the paddle of the impeller.

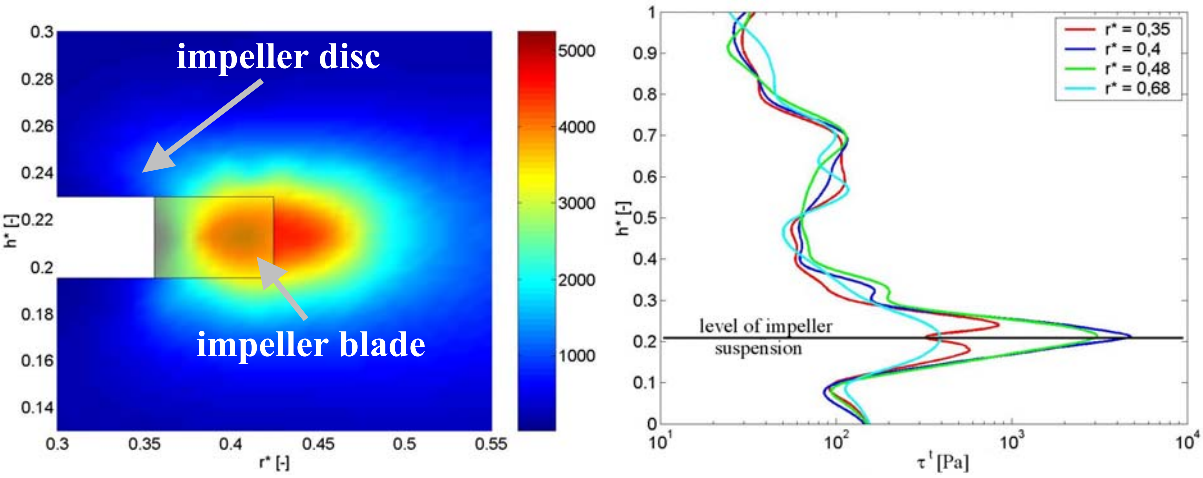

Fig. 4. Total turbulent stress τ^t [Pa]

All things considered, one may state that for turbulent flow the turbulent stress plays a key role in momentum transport. It must be added that the turbulent stress in the region of the impeller paddle is four orders greater than the viscous stress. That is why, it may be stated that at the onset of self-aspiration (with uncovered slots in the impeller shaft) when small amounts of the diffused phase do not alter the turbulence in significant manner in the region of the impeller paddle, the turbulent shear stresses tear off the gas portions from the interphase surface and break them into the smaller bubbles. This assumption is confirmed in experiments presented in the study [5].

4.2 Turbulent viscosity and turbulent diffusion coefficient

The equations (6) and (10) imply that the turbulent viscosity coefficient μ^t is a tensor quantity. In practice, no determination of all coordinates of the tensor is possible and, that is why, the simplified assumptions are often used. The typical example which allows to simplify the analysis of turbulent flow is the replacement of the tensor quantity (the variables in all directions of the coordinate system) with one scalar quantity. Such a simplification may be observed in the numeric models of turbulent flows. In a model k-ε [12] the value of the turbulent viscosity (the scalar quantity) is dependent on the energy dissipation rate ε as well as on the kinetic turbulent energy q

$$\mu^t = 0{,}09 \cdot \rho \cdot \frac{q^2}{\varepsilon} \tag{11}$$

and both quantities may be determined on the basis of the measurements of instantenous velocities [4, 13]. The calculations made using the equation (11) are presented in Fig. 5.

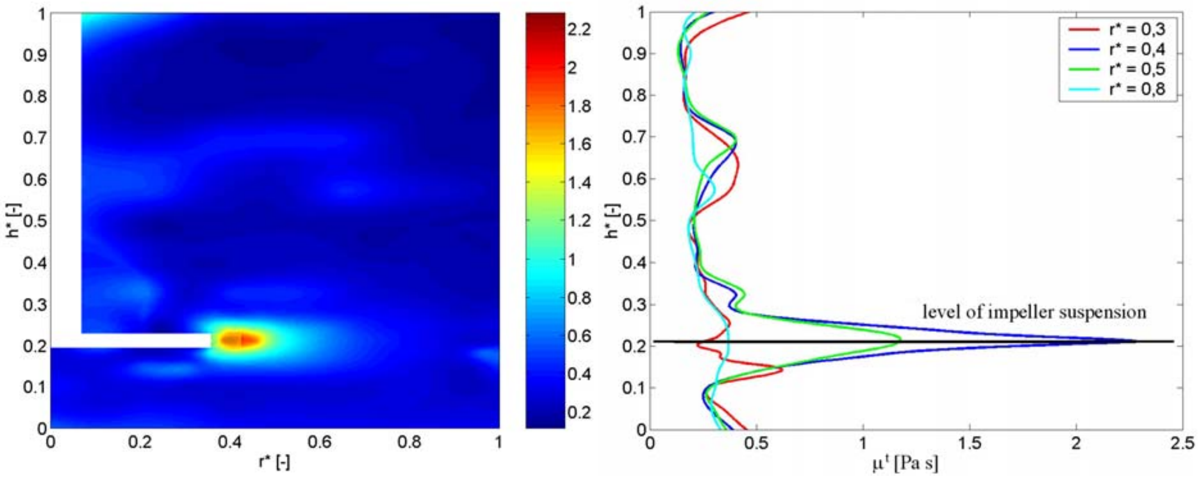

Fig. 5. Distribution of the local value of the turbulent viscosity μ^t [Pa·s]

As may be seen in the analysis of Fig. 5, the value of the turbulent viscosity μ^t in a developed turbulent flow is three orders higher than the molecular viscosity ($\mu=10^{-3}$ [Pa·s]). Hence, the influence of the molecular viscosity is limited to the zone close to the wall of the agitator. The mean value of the turbulent viscosity for the whole agitator was μ^t = 0,41 [Pas] which means that assuming in the equation (9) the value of the turbulent Schmidt number Sc^t = 1.0, the mean value of the turbulent diffusion coefficient in the agitator for the Reynolds number Re=95050 will be equal $D^t = 0.41\cdot10^{-3}$ [m²/s]. The results obtained are close to the results from the other papers [14, 15].

5. CONCLUSIONS

On the basis of the measurements and calculations one may draw the following conclusions:

- the highest viscous and turbulent stresses are present in the region of the impeller paddle,
- the turbulent stresses are three orders higher than the viscous stresses,
- the turbulent viscosity is three orders higher than the molecular viscosity,
- the mean value of the turbulent coefficient of diffusion for a self-aspiring impeller, for Re=95050, is $D^t = 0.41\cdot10^{-3}$ [m²/s].

NOMENCLATURE

c_A	- concentration of component A	[kg/m³]
p	- pressure	[Pa]
D_{Ay}, D^t_{Ay}, D^{ef}_{Ay}	- molecular, turbulent and effective diffusion coefficient	[m²/s]
v_x, v_y, v_z	- velocity components	[m/s]
q	- kinematic turbulent energy	[m²/s²]
μ	- dynamic viscosity [Pa·s]	
ν	- kinematic viscosity	[m²/s]
τ	- shear stress	[N/m²]
ε	- energy dissipation rate	[W/kg]

Indexes

x^*	- dimensionless value
x'	- fluctuating value
$\overline{x}$	- mean time value

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