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THE USE OF MOMENTUM RATIO TO EVALUATE THE PERFORMANCE OF CSTRS

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Abstract. In this study, the residence time distribution of a CSTR was measured and analysed for variations in CSTR feed geometry, impeller speed and inlet flow rate. The measured residence time distributions were also used to characterize the degree of plug flow behaviour and short-circuiting. The data showed that the performance of a CSTR could not be evaluated using the ratio of the mean residence time to the batch mode mixing time (τ/t_M), and that this approach can lead to process over-design and excessive energy requirements. Instead, it was shown that all process parameters can be correlated using the ratio of the inlet jet momentum to the impeller discharge momentum. The prediction of the degree of short circuiting and plug flow in the CSTR could be used to improve process control. In addition, the results can be used to identify whether the inlet position or diameter of an existing CSTR should be modified to improve process performance.

Key words: CSTR; Mixing; Residence time distribution; Scale-up; Momentum ratio

1. INTRODUCTION

Stirred tanks run under continuous flow conditions (CSTRs) are used extensively in the process industries as they can easily be integrated in a continuous process whilst providing long residence times at a relatively low capital cost. In addition, the backmixed nature of the CSTR's flow history is favourable for certain reactions and provides "smoothing" of time-varying fluctuation of product or reactant properties to aid process control [1].

A detailed analysis of continuous flow systems and the impact of non-ideal mixing behaviour on the process output were discussed in [2] and [3]. One rule of thumb for the design of CSTRs is to aim for "$\tau/t_M>10$". This means that the CSTR can be assumed to be a perfectly mixed system if the mean residence time is at least ten times the batch-mode mixing time [4]. It has been shown that when maintaining a constant residence time on scale-up, constant mixing time [5] or constant circulation rate [6] can be used. These two approaches are equivalent and similar to the use of a "$\tau/t_M > K$" approach [7].

τ/t_M has recently been used to present data on the degree of poorly-mixed and well-mixed zones in a CSTR with different impeller types [8]. However, the dimensionless group, τ/t_M, did not correlate the data presented for different impeller speeds. It was concluded that a trial-and-error approach was needed for CSTR design, where an impeller type and speed is selected and then an appropriate feed rate is chosen, based on the overall process requirements. This approach is impractical to implement in industry because the CSTR feed rate is usually fixed by the required rate of production in a manufacturing process, or by the size of the liquid flow to be processed in, for example, a water treatment works.

Another recent study has investigated the short timescale behaviour of a CSTR using conductivity probes in the CSTR and at the outlet to assess the degree of deviation from "perfect mixing" in a CSTR [9]. In this study, τ/t_M correlated the data better than an impeller-to-jet velocity ratio.

In terms of scale-up rules, using $\tau/t_M = 10$ and assuming that the residence time will be kept constant due to, for example, the reaction kinetics of the process, a constant impeller speed must be maintained. This can lead to unfeasible power requirements at large scale if small scale operating conditions are not selected carefully. Regardless of the feasibility of this approach for scale-up, $\tau/t_M = 10$ as a critical value is questionable. For example, a CSTR that meets the $\tau/t_M = 10$ criterion can be modified by adjusting the diameter of the inlet pipe. The velocity of the inlet jet would be changed and this would affect the short timescale flows in the CSTR and thus affect the residence time distribution, even though the ratio τ/t_M would remain constant. This point also illustrates an opportunity – if a modification to the inlet can be used to lower the impeller speed, an optimised CSTR design with lower power requirements can be developed. A CSTR design with a value of $\tau/t_M = 5$ would have an eightfold lower energy requirement compared to a system designed for $\tau/t_M = 10$.

This study has been carried out to challenge this "rule of thumb" and propose a different approach for characterising CSTRs.

2. EXPERIMENTAL SET-UP AND DATA ANALYSIS

The experimental set-up is shown in Figure 1. The impeller used was a 4-bladed PBT with a blade angle of 45°. For this study, different inlet pipe diameters of either 10 or 25mm were used. The inlet was positioned 50mm ($T/12$) below the liquid surface, pointing horizontally towards the stirrer shaft. The outlet was on the opposite side of the tank, with the centre in line with the impeller centre line. The inlet flow rate was measured using an in-line calibrated rotameter. The discharge flow rate and liquid level were controlled using an overflow weir downstream of the CSTR outlet.

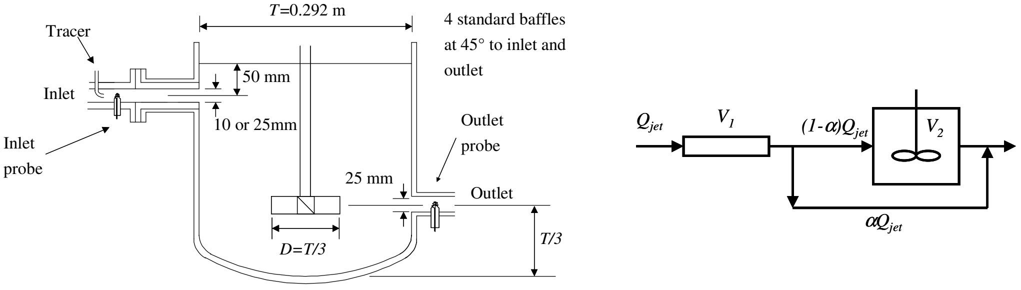

Fig. 1. Diagram of the CSTR geometry, and the flow model used for data fitting.

The residence time distribution was measured using a salt solution injected upstream of the pipe inlet. Conductivity probes were positioned in the inlet and outlet of the CSTR. The lag time of the inlet and outlet pipe-work was measured using additional conductivity probes and subtracted from the experimental data. The conductivity probes and associated hardware are described in [10] Further details on the experimental set-up and techniques can be found in [11].

3. DATA ANALYSIS

The analysis performed in this work required integration of the probe responses from time 0 to infinity. This is not feasible with experimental time series data, both because of the finite resolution of the analogue to digital converters and the difficulty in collecting data over extremely long times with no variation in the CSTR feed properties. Therefore the data were processed to generate an analytical expression for the "tail" of the outlet tracer concentration. The constants used in this analytical solution were obtained from a non-linear regression on a portion of the collected experimental data [11].As well as calculating the mean residence time for comparison with the inlet flow rate data, the data were processed to give the exit age distribution, the variance of the exit age distribution and the lag time (the time for first detection of tracer in the outlet flow).

The variance of a perfectly back-mixed system can be shown to be equal to 1. A degree of plug flow behaviour will tend to narrow the residence time distribution and thus reduce the variance. The most extreme example of this would be perfect plug flow, which would give a variance of 0. When short-circuiting occurs, there is a direct contribution to the variance from the early-exiting material. In addition, the fractional flow through the well-mixed region will be reduced, which will broaden the rest of the exit age distribution of the CSTR. These two factors increase the variance of the CSTR to a value of more than one.

Under conditions where there is a certain degree of short-circuiting and also some plug flow behaviour, it is possible to obtain a variance of 1, despite non-ideal mixing. In order to fully assess the flow within the CSTR and characterise it in terms of plug flow and short-circuiting, the "residual" curve was developed. The difference between the expected (ideal) behaviour and the measured exit probe response was measured and integrated up to different values of time, t. Any plug flow behaviour will lead to a smaller than expected amount of tracer leaving the CSTR over short time periods and this gives a negative contribution to the residual curve. Short-circuiting behaviour will lead to some tracer leaving the CSTR too early, which gives a positive overall contribution to the residuals curve, at the point in time when the fluid reaches the outlet. Notwithstanding cases where material is either consumed or trapped in stagnant regions, the residual curve always tends to a value of 0 at large values of t because all of the fed material should eventually leave the CSTR, irrespective of the flow patterns. An example of a residence time distribution curve and the derived residuals curve is shown in Figure 2.

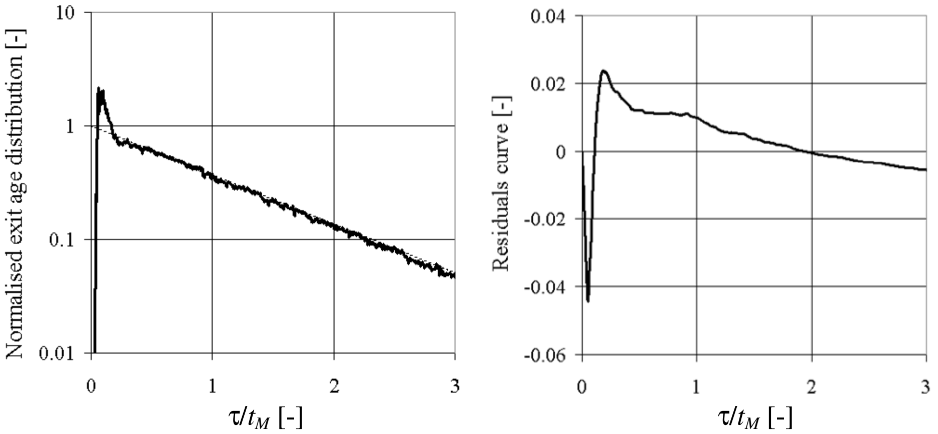

Fig. 2. Normalised exit age distribution and the derived residuals curve showing both a degree of plug flow behaviour and short-circuiting.

A number of dimensionless groups can be derived as possible correlating groups to evaluate the performance of a CSTR, including kinematic ratio (tip speed/inlet jet velocity), jet/impeller kinetic energy, momentum ratio and impeller/jet Reynolds number [11]. For brevity, data will only be presented for the timescale ratio, τ/t_M, and the impeller-to-jet momentum ratio. It can be shown that the Reynolds number ratio will give the same result as for the momentum ratio. However, the momentum ratio has a clearer basis for its applicability: the short timescale jet behaviour should depend on the interactions of two liquid flows which should be characterized by the relative magnitude of their momentum fluxes.

The timescale ratio, τ/t_M, can be calculated using the FMP mixing time correlations [12]:

$$\frac{\tau}{t_M} = \frac{V}{Q_{jet}} \times \frac{Po^{1/3} N D^2}{5.47 T^{1.5} H^{0.5}} \tag{1}$$

The momentum ratio will be:

$$\frac{Mo_{imp}}{Mo_{jet}} = K \frac{N^2 D^4}{Q_{jet} V_{jet}} \tag{2}$$

A value for K=1985 was derived from LDA data for a $D=T/3$ pitched blade turbine identical to that used in this study [13].

4. FLOW MODEL FOR DATA ANALYSIS

The approach used in this work was a variation of the plug flow and short-circuiting models used by [2] and [14], and is shown in Figure 1. No "dead" volume was used in the CSTR model because all measurements were performed at impeller Reynolds numbers of above 5000 and, therefore, no stagnant regions should be present in the CSTR. This was confirmed by comparing the measured and calculated mean residence times for the CSTR.

Using the model shown in Figure 1 with a bypass fractional flow rate, α, and a fractional plug flow of $(V_1)/(V_1+V_2)$, then the expression for the short-circuiting peak would be:

$$C(t) = \alpha \delta \left(\frac{t - t_{lag}}{\tau} \right) \tag{3}$$

where δ is a Dirac delta function, and the remaining tail due to the well-mixed region is:

$$C(t) = \alpha \delta (1 - \alpha) \frac{Q_{jet}}{V_1} e^{-(1-\alpha)(t - t_{lag}) \frac{Q_{jet}}{V_2}} \tag{4}$$

For normalised data, such that $V/Q_{jet}=1$, the variance will be:

$$Variance = \frac{V_1^2}{Q_{jet}^2} + \frac{2V_1V_2}{Q_{jet}^2} + \frac{2V_2^2}{(1-\alpha)Q_{jet}^2} - 1 \tag{5}$$

and the "residual" equation for the data is generated by calculating:

$$R(t) = \int_0^t (C - C_{theo})dt = \int_0^t (C - C_0 e^{t/\tau})dt \tag{6}$$

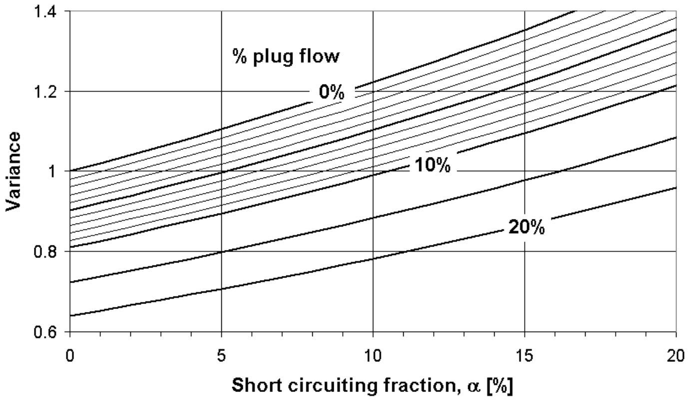

Fig. 3. Variance as a function of short-circuiting and plug flow, based on the model in Fig. 1.

Based on these equations, Figure 3 shows variance as a function of the amount of short-circuiting, for different degrees of plug flow in a CSTR.

5. FLOW VISUALISATION

Qualitative visualisation of the flow patterns and jet/impeller interactions in a CSTR was performed using dye additions and with a 25mm diameter inlet. Pulses of dye were injected upstream of the inlet to show the short timescale history of the inlet jet. Figure 4a shows the CSTR behaviour when the jet is dominant. The inlet jet flows across the top of the tank and then down towards the exit pipe, leading to a degree of short circuiting. The time taken for the jet to flow across the tank and down to the exit gives the lag time for the CSTR.

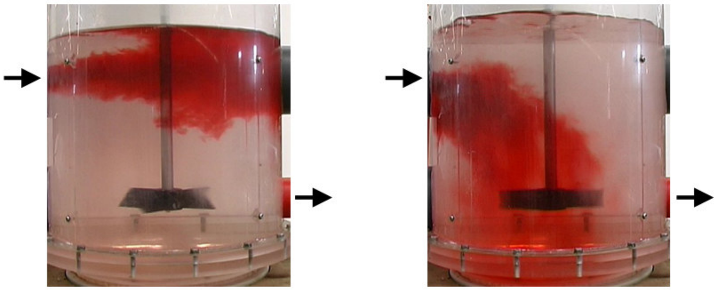

Fig. 4. Short time-scale jet behaviour for (a, left) a jet dominated case (0.3 l/s, 30 rpm), and (b, right) an impeller dominated case (0.3 l/s, 300 rpm)1.2 seconds after the addition of a dye tracer

Figure 4b is an example case where the impeller flow is dominant. The inlet jet flows into the impeller suction and is subsequently better mixed and distributed throughout the CSTR. In all cases, the jet and impeller flow interaction depended on both jet flow rate, which is related to mean residence time, and impeller speed, which is related to the batch mode mixing time.

6. DISCUSSION OF RTD RESULTS

One would expect that the residence time distribution would depend on the inlet pipe diameter in addition to the impeller speed, diameter and inlet flow rate. Therefore, the residence time distributions were measured whilst varying these operating conditions. The variance, lag time, short-circuiting and residual values were derived from the residence time distribution data and the average values from five experiments are presented here.

Figure 5 shows the variance as a function of τ/t_M for a 25mm diameter inlet jet. Three different inlet flow rates were used and tests were performed at varying impeller speeds to obtain the different values of τ/t_M. Tests at $\tau/t_M < 1$ also showed variance values close to 1.

The flow visualisation results showed highly non-ideal behaviour. Therefore it is expected that these results would be due to both short-circuiting and long lag times in the CSTR. At a slightly higher $\tau/t_M=2$, the variance increased to a maximum of 1.15-1.2. A comparison with Figure 3 shows that this corresponds to at least 7% short-circuiting in the CSTR, depending on the lag time. As the mixing rate was increased, the variance decreased and eventually dropped to a value of about 0.95 for $\tau/t_M=7$. This corresponds to very low short-circuiting and a finite and measurable lag time in the system. At $\tau/t_M>10$, the system appears to be perfectly mixed (within the experimental error).

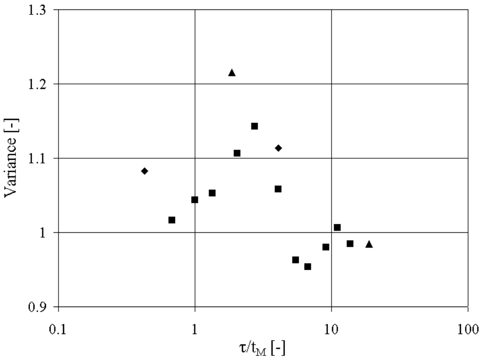

Fig. 5. Variance plotted as a function of τ/t_M for D_{jet}=25mm.

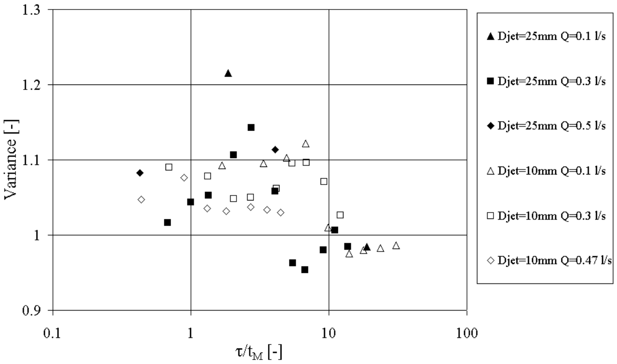

Fig. 6. Variance data for two different pipe diameters

Figure 6 shows data for two different inlet pipe diameters. The data for different flow rates and impeller speeds have been combined into a single data set. It is clear that there are three separate trends on the graph and thus the τ/t_M relationship does not correlate the data. The CSTR with the larger diameter inlet jet shows "ideal" behaviour at a lower value of τ/t_M, and the smaller diameter jet (D_{jet}/T~30) non-ideal behaviour at $\tau/t_M>10$. Overall, however, the trend of increasing and then decreasing variance with increasing τ/t_M can be seen for all three sets of data.

Figure 7 shows the same data plotted with variance as a function of the impeller to jet momentum ratio. The momentum ratio group has collapsed the data for the D_{jet}=10mm and D_{jet} =25mm cases. All data show the characteristic short-circuiting at intermediate momentum ratio and show also that the CSTR appears to perform well at momentum ratios of above about 80-100.

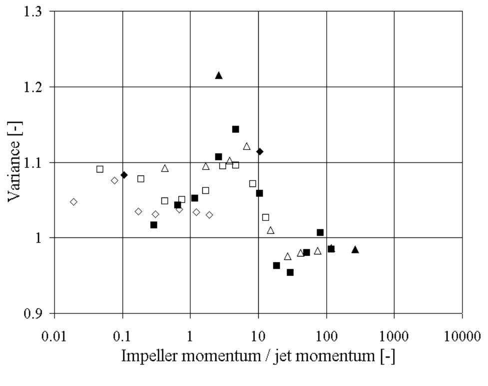

Fig. 7. Variance as a function of momentum ratio.

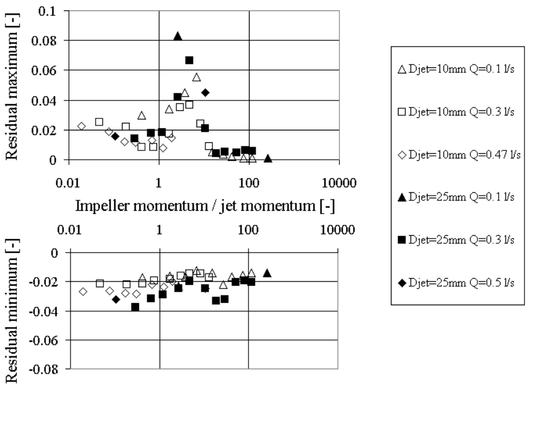

Fig. 8. Maximum and minimum values of the residuals curve for different operating conditions.

The maximum and minimum values of the residuals curves are shown in Figure 8. At momentum ratios of below 1, the residuals curves indicated some plug flow behaviour and some short-circuiting. At momentum ratios of between 1 and 10, the higher degree of short circuiting could be seen, which also led to the increased measured variance. At momentum ratios of above 10, there was less short circuiting, but there was still a measurable degree of plug flow behaviour. At momentum ratios of above 100, the degree of plug flow behaviour was reduced further.

The data were analysed further to derive the degree of short-circuiting and plug flow, and these results are shown in Figure 9a and 9b. The results confirm the observations from the residuals curves, and in particular they show that the degree of plug flow varies only slightly with changes in the momentum ratio, but that the degree of bypassing has a large impact on the system variance.

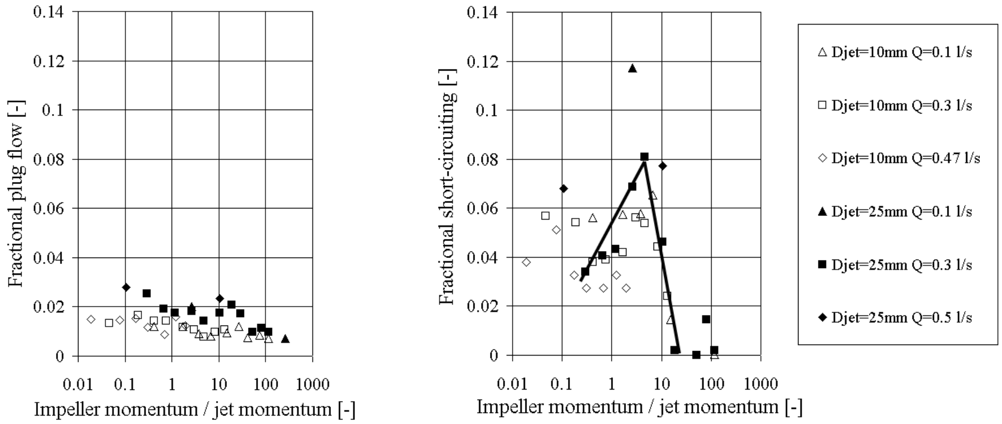

Fig. 9. Fractional plug flow and fractional bypassing as a function of impeller to jet momentum ratio.

This information on the degree of short-circuiting and plug flow could be useful for improved tuning of process control systems. The information can also be used to assess whether an existing CSTR will be prone to short-circuiting and to predict the effect of modification of the inlet in order to improve its performance. For example, if an existing CSTR was designed for τ/t_M=10, but the chosen inlet diameter were too small, the momentum ratio would be less than 100 and severe short-circuiting could occur. The momentum ratio could be improved either by changing the impeller rotational speed or by changing the diameter of the inlet jet. An increase in the impeller speed would lead to higher energy input and may require a new gearbox and new electrical motor. An increase in the inlet pipe diameter in order to increase the momentum ratio would lead to lower pipe pressure drop and thus would reduce the plant operating costs, whilst also avoiding Capital outlay.

7. CONCLUSIONS

In this study it has been shown that the dimensionless group, τ/t_M, does not correlate data for CSTRs with varying inlet diameters, whereas a new approach based on the ratio of the impeller to jet momentum can successfully be used. For the geometry tested, a momentum ratio of above 100 was required to achieve a “perfectly mixed” CSTR.

The CSTR geometry tested was not designed to have high degrees of short-circuiting, and the inlet and outlet were on opposite sides of the tank, a recommendation given in [9]. Nevertheless, the residuals data showed that up to 8% short-circuiting can occur at intermediate momentum ratios. In CSTRs where short-circuiting can occur, an increase of the inlet pipe diameter would be more energy efficient than a higher impeller speed.

For scale-up of geometrically similar CSTRs with constant mean residence time, either constant momentum ratio or constant τ/t_M approaches can be used. These are both equivalent to keeping impeller speed constant on scale-up. Small scale experiments should be run at low impeller speeds to make sure that the operating conditions can be replicated at large scale.

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NOMENCLATURE

t_{lag}	Lag time, *s*
$R(\mathrm{t})$	Residual function
Q	Volumetric flow rate, m^3s^{-1}
α	Fractional degree of bypassing in CSTR
β	Fractional degree of plug flow in CSTR
τ	Mean residence time, *s*
Mo	Momentum flux of either impeller or inlet jet, $kgms^{-2}$
imp, jet	Subscripts for impeller and jet flows

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