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EVALUATION OF LOCAL KINETIC ENERGY DISSIPATION RATE IN THE IMPELLER STREAM OF A RUSHTON TURBINE BY TIME-RESOLVED PIV

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Abstract. The present paper is dedicated to the direct measurement of the dissipation rate of kinetic energy by means of time-resolved 2D-PIV in the impeller region of a stirred vessel. Experimental estimation of $\overline{\varepsilon}$ is carried out without simplifications, since all spatial gradients of the fluctuating velocity components are directly measured. At each point, measurements have thus been realized in three orthogonal planes. The present results have been compared with previous ones and discussed according to experimental techniques. A special attention is focused on the contribution of measurements in each plane and some new ideas have been suggested for the assessment of $\overline{\varepsilon}$. In addition, following each measurement plane, the time-variation of the instantaneous fluctuating velocity gradients have been also presented, exhibiting intermittency, with large instantaneous values linked to the trailing vortex induced by the blade rotation.

Key words: Stirred Tank; Time-resolved PIV; Kinetic Energy Dissipation; Heterogeneity

1. INTRODUCTION

Mixing investigation in stirred tank reactor has retained a lot attention in the last twenty years [1, 2]. Hydrodynamics and in particular the turbulent flow properties play a crucial role in chemical or biochemical processes involving mixing at the molecular scales [3]. One of the main scopes in the chemical engineering area is the estimation of the magnitude of the turbulent kinetic energy dissipation rate, $\overline{\varepsilon}$ since it controls the rate of mixing at the molecular scales [4]. Thus, the local distribution of $\overline{\varepsilon}$ in the stirred tank will definitely allow a better prediction of the processes in view of their optimization and up-scaling. One of the main issues of the turbulent kinetic energy dissipation measurement is that it corresponds to the smallest eddies participating to the mixing. Despite the tremendous increase computer capacities, it remains difficult to access to $\overline{\varepsilon}$ because of the needs to solve the whole range of the turbulent scales (from Taylor macroscales down to Kolmogorov microscales). Up to now, direct numerical simulations remains limited in complex geometries such as baffled mixing tanks. Most experimental studies largely cited in some review paper [5] have been performed using hot wire anemometry or laser Doppler anemometry (LDA) [6] where $\overline{\varepsilon}$ is calculated in one or two point by means time series analysis or spatial correlation technique. In other hand, most of the authors assumed the local isotropy of the turbulence in spite of strong heterogeneity characteristics [7] due to the trailing vortex. More recently, the use of PIV technique [8] showed promising results to access to $\overline{\varepsilon}$ by spatial fluctuating velocity gradient measurement. In some papers using PIV [9, 10], authors have estimated $\bar{\varepsilon}$ from an experimental balance of the kinetic energy equations based on a triple decomposition of the instantaneous velocity in order to distinguish mean flow kinetic energy, periodic flow kinetic energy from purely turbulent kinetic energy. Periodic flow components were mainly related to trailing vortices. The authors have demonstrated that the mean flow and the periodic flow contributions to the total dissipation were negligible, the turbulence dissipating all the kinetic energy. However, they have showed that a strong exchange of kinetic energy occurred at the trailing vortices locations. From these last results, Baldi and Yianneski (2004) [11] proposed to measure directly the fluctuating velocity gradient in order to obtain the turbulence energy dissipation by 2D PIV, limited to a low frequency acquisition. Thus, they were able to measure five terms among the twelve terms of the turbulence energy dissipation. The authors have used the local isotropy hypothesis [12] to identify the seven terms unknown. They assumed that the radial and tangential turbulence levels are more similar in magnitude than the axial ones. These hypotheses allowed the estimation of $\bar{\varepsilon}$ with only one measurements plane. Notwithstanding these hypotheses, they found comparable results to those expressed in literature and concluded that the direct access to the twelve terms of the kinetic energy dissipation rate would constitute a promising challenge in order to improve the understanding of mixing processes. In the present work, we propose to measure all the velocity gradients in order to obtain the real value of $\bar{\varepsilon}$. One of the definitions of $\bar{\varepsilon}$ [13] may be given in tensorial form by writing:

$$\bar{\varepsilon} = 2\nu\overline{\left(s'_{ij}\, s'_{ij}\right)} \qquad (1)$$

$$\text{with}\, s'_{ij} = \frac{1}{2}\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right) \qquad (2)$$

The measurements are performed with high frequency PIV at three locations in the impeller stream (section 2) of a Rushton turbine. At each location, measurements are performed in three orthogonal planes. For each measurement plane, the calculation of all the components of the fluctuating velocity gradient is carried out. The measurements presented here are not angle-resolved since only the turbulence dissipates kinetic energy [9]. The results are first validated by comparison with the literature data (section 3.1) and each measurements plane is analysed in terms of mean and instantaneous values. New assumptions are proposed for the evaluation of kinetic energy dissipation rate from only one measurement plane (section 3.2). In addition, the heterogeneity of dissipation rate due to the flow characteristics linked to the blade rotation are discussed (section 3.3). Conclusions are finally drawn (section 4).

2. EXPERIMENTAL SET-UP AND METHODOLOGY

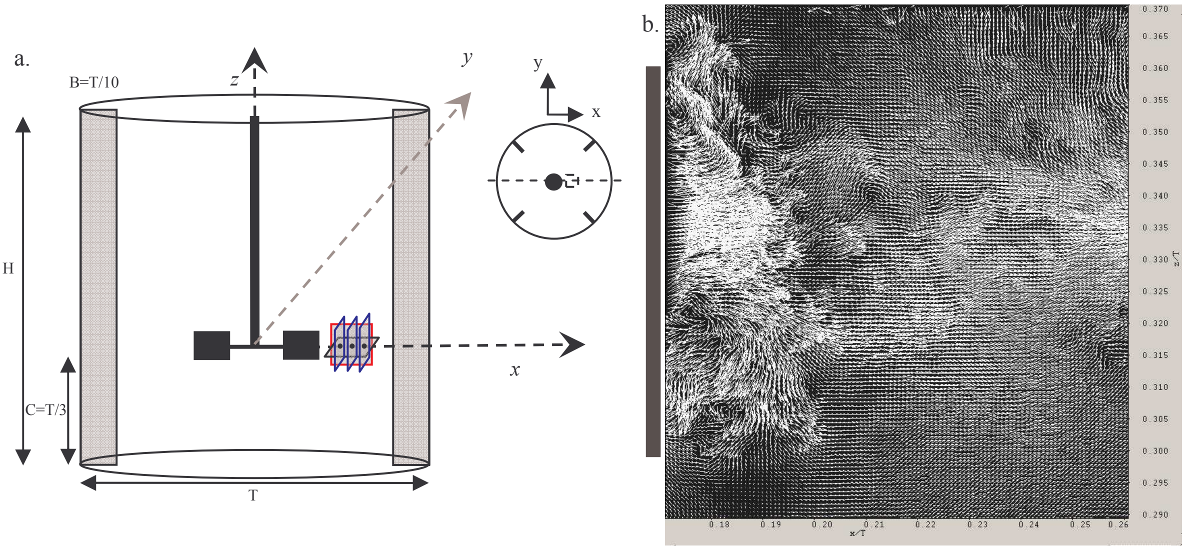
2.1 High Frequency 2D PIV

The time-resolved 2D-PIV enables instantaneous measurements of velocity field without filtering the high frequency fluctuations. The PIV system, provided by Lavision, includes a double-pulsed Nd-YLF Newwave laser Pegasus (2×10 mJ at 1KHz) and a Photron CMOS RS3000 camera (3kHz at a resolution of 1024 × 1024 pixels). The thickness of the laser sheet was kept as low as possible (400 µm). Rhodamines particles were used as seeding particles and their average sizes was less than 20 µm (density=1.1). A dedicated processor, Davis 7.1, was used to acquire data and thus to process data and perform the calculations of flow fields. The velocity field has been analysed by adaptative cross-correlation 50% overlapping windows with interrogation cell sizes of 32 pixels squared for the preliminary step and 16 pixels squared for the high-resolution. The camera was focused on a 35 mm×35 mm area, using a 105 mm objective with a diaphragm aperture of f/8 and the size of the 16 pixels interrogation area corresponds to 300 µm indicating the spatial resolution, ΔZ. For each set of 2D-PIV measurements, 6144 pictures were taken at a frequency of 2000 Hz and the measurements were repeated three times to attest reproducibility.

2.2 Experimental apparatus

The 2D experiments were carried out in a transparent vessel of diameter T=450 mm, stirred by a Rushton Turbine of diameter equal to D=T/3 with six blades of thickness T/150. A standard configuration was employed with vessel height H=T and four equi-spaced baffles of width B=T/10. The clearance, C, was equal to the impeller diameter and was measured between the bottom of the mixing vessel and the impeller-disk plane. The blade height is equal to 0.2 D and the disk and the blades are the same thickness. The orientation of the Cartesian reference frame is plotted in the fig. 1a. x corresponds to the radial direction (positive toward the tank wall), y the tangential direction and z the vertical direction.

Fig. 1: a. Vertical and top view of the measurements planes
b. Times-resolved flow field in the impeller stream (xz-plane)



The experiments were carried out at the impeller rotation speed of N=50 rpm. The Reynolds number ($Re=ND^2/\nu$) is equal to 18750 and the flow regime is turbulent. The power number is defined as:

$$N_p = \frac{P}{\rho N^3 D^5} \tag{3}$$

The value of N_p is constant in the turbulent flow regime and equal to 5.5. It correspond to a power, P=0.24 W in our rotation speed condition. Thus, the averaged volumetric dissipation rate in the stirred tank, $\langle \varepsilon \rangle$, is given by:

$$\langle \varepsilon \rangle = \frac{P}{\rho V} = 0.0033 \ \mathrm{m^2.s^{-3}} \tag{4}$$

From this value, one can approaches the mean value of the Kolmogorov microscales, $\langle \eta \rangle$, according to the following expression:

$$\langle \eta \rangle = \left(\frac{\nu^3}{\langle \varepsilon \rangle} \right)^{1/4} \tag{5}$$

This first evaluation of the Kolmogorov microscales is primordial for the accuracy of the PIV measurement. As mentioned by Sharp and Adrian (2001) [8], strong attention has to be focussed on the spatial resolution which must be similar to the Kolmogorov length scales, in order to take into account the smallest turbulent eddies present in the flow, involved in the turbulent dissipation. Under our experimental conditions, the spatial average of the Kolmogorov length scales in the vessel is equal to about 130 μm corresponding to a ratio $\Delta Z/\langle\eta\rangle$ less than 3. This limit value of 3 has been reported by Saarenrinne and Piirto (2000) [14] as the limit value of $\Delta Z/\langle\eta\rangle$ for a good evaluation of the dissipation, with an error range equal to 5 %. Authors have suggested that flow with large turbulent microscales will involve more reliable results.

Three locations at z/T=0.33, shown in fig. 1a, are chosen to describe the turbulent energy dissipation where the magnitudes of the ε were found to be larger [9] [x/T=0.211, x/T=0.222, x/T=0.233]. The angular position is set between two baffles at θ=45°.

Five measurements planes have been found necessary for the determination of the three components of the fluctuating velocity and the twelve components of the turbulent energy dissipation at the three locations. A radial one (xz-plane) and a horizontal plane (xy-plane) in order to calculate respectively the components u,w and u,v and their gradient. Three tangential planes (yz-plane) were also performed in order to calculate the components v and w and their gradient.

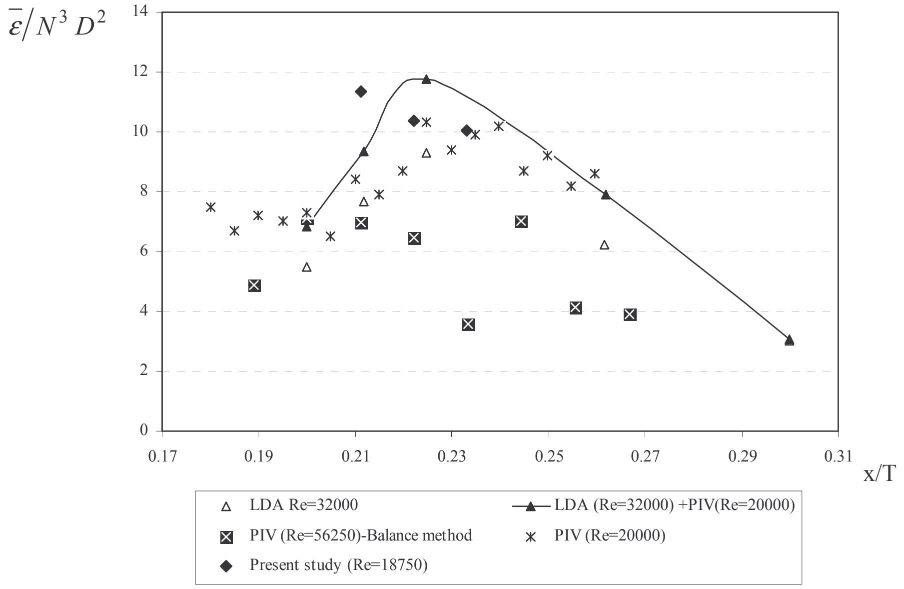
The result showed in fig. 1b presents an instantaneous flow field in the impeller stream in the xz-plane at the blade passage. The centre of the disk being located at z/T=0.33, it results at this level a radial jet with high velocity on the left side of the picture close to the impeller and lower values away the blade. In other hand, one may distinguish vortices appearing at the blade passage and advected radially in the main direction flow, x/T. By comparison with the mean velocity fields largely presented in the literature [15], the instantaneous flow fields differ remarkably. These heterogeneous instantaneous characteristics proved the strong interest to study locally the energetic behaviour of the turbulent motion.

3. EXPERIMENTAL RESULTS

3.1 Comparison with previous results

Dissipation rate of the kinetics turbulent energy is shown in fig. 2 and compared with others technique as LDA and classical PIV [16, 17].

Fig. 2: : Kinetic energy dissipation in the impeller stream obtained with different measurements techniques



Our results are close to those of Ducci and Yianneskis (2005), realized with LDA [17] with larger Reynolds number, despite a larger spatial resolution in our case and balanced by a smaller Reynolds number. Thus, in the two studies, the ratio $\Delta Z / \overline{\eta}$ has the same order of magnitude. The dimensionless values of the dissipation rate $\overline{\varepsilon} / N^3 D^2$ are in good agreement with the other studies. One can notice that the previous method performed in the same tank [9] and based on turbulent kinetic energy balance underestimates the dissipation rate. The peak value, equal to 11.3, is estimated at r/T=0.21 in the present case study instead of r/T=0.23 reported by Ducci et al. (2004) [15] and Baldi et al. (2004) [11]. It may be attributed to the experimental conditions, such as slight difference of blade thickness or vertical positioning of the impeller, which may be considered sufficient to cause significant difference in the results.

3.2 Contribution of each plane in the assessment of the turbulent dissipation

For an instantaneous measurement of ε, the measurements should be carried out simultaneously in the three planes. This has not been the case in the present study. Thus, we propose to decompose the turbulent kinetic energy dissipation rate, $\overline{\varepsilon}$ in three components corresponding to the three measurements planes. In each plane, we introduce specific contribution to the total dissipation rate, $\overline{\varepsilon_{xz}}$, $\overline{\varepsilon_{xy}}$ and $\overline{\varepsilon_{yz}}$ defined as:

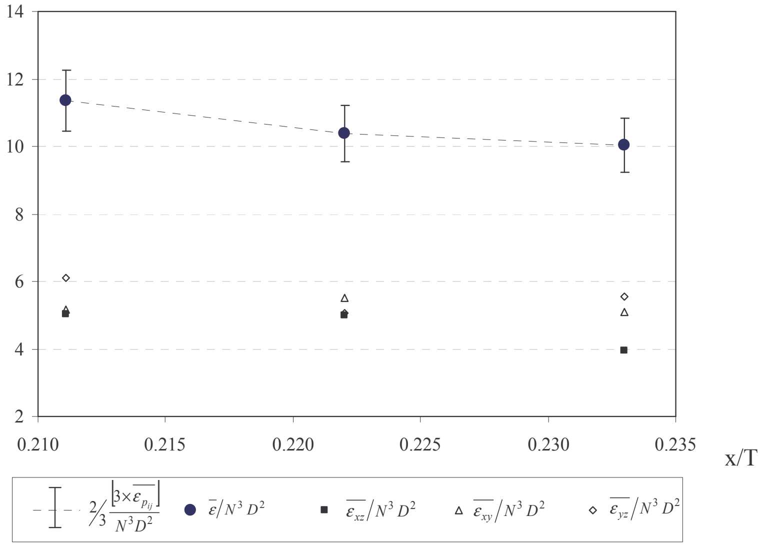
$$\overline{\varepsilon_{p_{ij}}} = \nu \left\{ 2\overline{\left(\frac{\partial u_i}{\partial x_i}\right)^2} + \overline{\left(\frac{\partial u_i}{\partial x_j}\right)^2} + \overline{\left(\frac{\partial u_j}{\partial x_i}\right)^2} + 2\overline{\left(\frac{\partial u_j}{\partial x_j}\right)^2} + 2\overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}} \right\} \tag{6}$$

with p_{ij}, the measurement plane xz, xy or yz. Thus, the turbulent kinetic energy dissipation rate $\overline{\varepsilon}$ can also be deduced by:

$$\overline{\varepsilon} = \overline{\varepsilon_{xz}} + \overline{\varepsilon_{xy}} + \overline{\varepsilon_{yz}} - \nu \left\{ 2\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 2\overline{\left(\frac{\partial v}{\partial y}\right)^2} + 2\overline{\left(\frac{\partial w}{\partial z}\right)^2} \right\} \tag{7}$$

Fig. 3 compares the contributions of each plane, compared to the total turbulent kinetic energy dissipation rate. It can be observed that each contribution is approximately identical, confirming the isotropy assumption proposed by most authors [10].

Fig. 3: Contribution of each measurements plane in the assessment of the turbulent energy dissipation



At these three points, the turbulent kinetic energy dissipation rate can be also deduced by the following expression with a standard deviation of 8 % whatever the measurement plane:

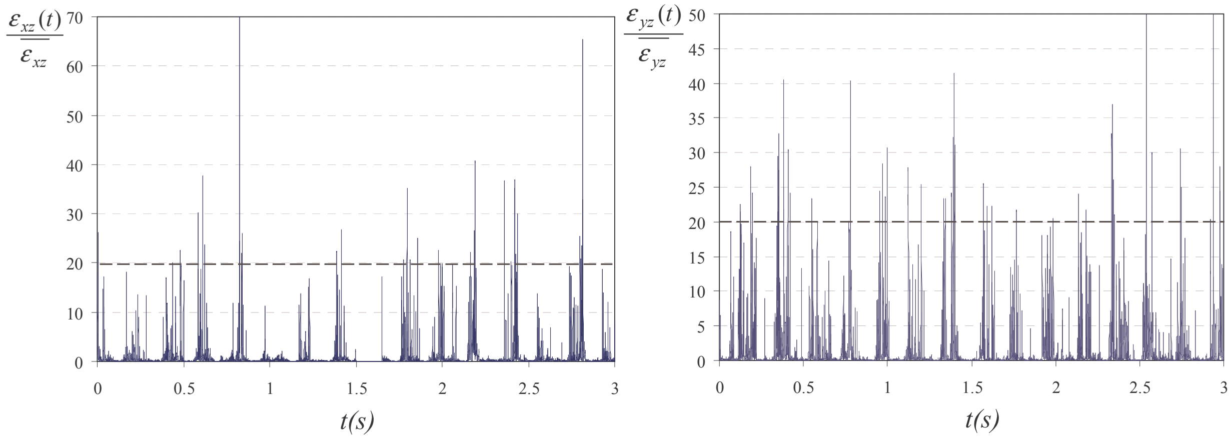
$$\bar{\varepsilon} = \frac{2}{3}(3 \times \overline{\varepsilon_{p_{ij}}}) \tag{8}$$

Finally, we propose to analyse the dimensionless time-variation of $\varepsilon_{xz}(t)$ and $\varepsilon_{yz}(t)$ recorded during three seconds and plotted in fig. 4.

The instantaneous results exhibit periodic behaviour. When we examine the graphics, 15 periods can clearly be observed at regular interval. For every period, two sequences corresponding to large and low values of instantaneous dissipation can be observed. It is clear that this periodicity corresponds to the blade passage frequency (f=6N). In 3 seconds, the impeller executed 2.5 rotations; since the impeller has 6 blades, it corresponds to 15 periods. It is interesting to observe that high dissipation rate corresponds to trailing vortex regions. This periodic structure has also been confirmed by Fourier analysis applied to the data.

The magnitude of the instantaneous values, located in the trailing vortices, reached up to 20 times the turbulent kinetic energy dissipation rate averaged in one plane. One can attribute larger values ($\varepsilon_{xz}(t) > 20\overline{\varepsilon_{xz}}$) to the dissipation rate noise induced by over-estimated velocity fluctuation gradients during the blade passage. Anyway, the trailing vortices seem to correspond to larger values of dissipation rate of the kinetics turbulent energy. This result is coherent with the findings of Escudié and Liné (2003) [9] who revealed that a significant transfer of kinetic energy occurred in the trailing vortices, between periodic flow and turbulence.

Fig. 4: Time variation of the contribution of the energy dissipation in the xz- and yz-plane at x/T=0.21



3.3 Instantaneous characteristics of the turbulent energy dissipation in the impeller stream

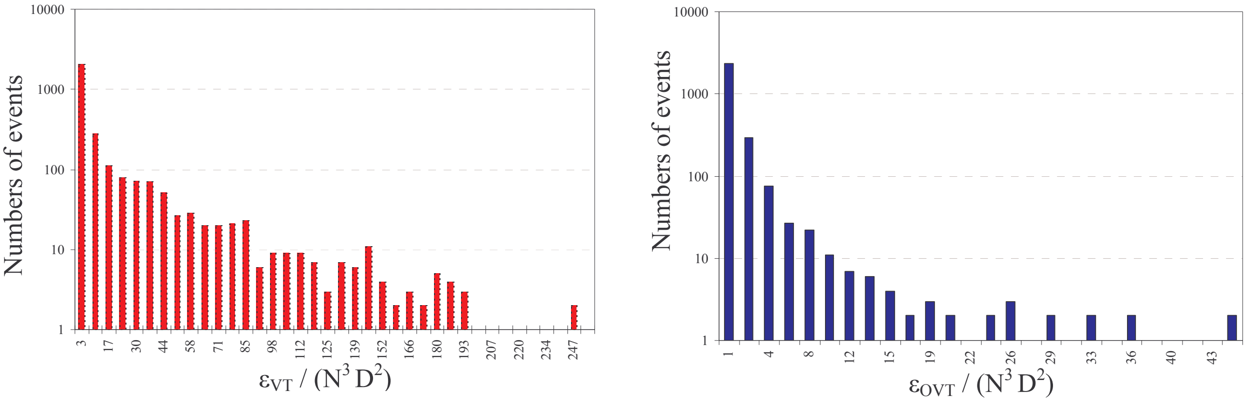
In this last section, we propose to focus the study on the instantaneous values of the kinetic energy dissipation rate. In order to describe the time-variation of the turbulent kinetic energy dissipation rate linked to the frequency of successive blades passage (f=5Hz), a criterion similar to a Heaviside function is applied. It corresponds to the time passage of trailing vortex, Δt_{VT}, where the instantaneous values are found to be large. The remaining time, Δt, corresponds to the lower values of the kinetic energy dissipation rate. Thus, the contribution of the time, β, corresponding to the large values of $\varepsilon(t)$ is given by the following expression:

$$\beta = \frac{\overline{\Delta t_{VT}}}{\overline{\Delta t_{VT}} + \overline{\Delta t}} \tag{9}$$

This procedure is applied for each measurement plane allowing extraction of two components of $\varepsilon_{p_{ij}}(t)$ showed in fig. 5 in the yz-plane. One is named ε_{VT} corresponding to the variation of

$\varepsilon_{plane}(t)$ inside the trailing vortex. The second one, ε_{OVT}, corresponds to the variation of $\varepsilon_{p_{ij}}(t)$ outside the trailing vortex.

Fig. 5: Decomposition of ε_{yz} (t) / ($N^3 D^2$) in two components:
a. Outside of the trailing vortex. b. Inside the trailing vortex.



The recapitulative results for the three measurement planes are gathered in the table 1.

Table1 : Recapitulative results for each plane

Plane p_{ij}	xz	xy	yz
$\overline{\varepsilon_{p_{ij}}} / N^3 D^2$	5.04	5.17	6.10
$\overline{\varepsilon_{OVT}} / N^3 D^2$	1.96	3.16	1.47
$\overline{\varepsilon_{VT}} / N^3 D^2$	7.43	7.51	13.56
β	0.51	0.51	0.52

The same trend appears for each plane with a value of β close to 0.5. Thus, the half time-recording corresponds to the largest values of $\varepsilon(t)$ which can be decomposed by the following expression:

$$\bar{\varepsilon} = \overline{\beta}\,\overline{\varepsilon_{VT}} + (1-\overline{\beta})\overline{\varepsilon_{OVT}} \qquad (10)$$

$\overline{\beta}$ being the time-contribution of trailing vortices to $\varepsilon(t)$. In our experimental condition at N=50 rpm, this value is equal to 0.5 with a mean standard deviation equal to 10 %. These heterogeneous phenomena of the dissipation rate induce variations of the Kolmogorov microscales, with smallest values located in the impeller stream, during trailing vortex passage.

4. CONCLUSION

The time-resolved 2D-PIV was performed in the impeller stream of a Rushton turbine. The experiments were performed in three measurement planes in order to measure all the spatial derivatives of the fluctuating velocities. Thus, a complete estimation of the dissipation rate of the turbulent kinetic energy was performed, without any turbulence assumptions. The present results have been compared with previous experimental study. These results are in good agreement with the other measurement techniques using different procedures for the assessment of $\bar{\varepsilon}$. In addition, the contribution of each measurement plane in the assessment of $\bar{\varepsilon}$ has been considered from the mean and instantaneous values, exhibiting several characteristics. The local isotropic turbulence assumption was confirmed. The dissipation rate of the turbulent kinetic energy can be evaluated from only one plane $\varepsilon_{p_{ij}}(t)$. An original result is that instantaneous dissipation rate exhibits periodicity, with large values located in the trailing vortex.

Finally, a multi-scale decomposition of the dissipation rate of the turbulent kinetics energy is proposed according to the following parameter: $\langle \varepsilon \rangle$ at the scale of the reactor, $\overline{\varepsilon}$ depending on the location in the tank, especially in the impeller stream where the flow is more complex and where its value can be decomposed in two contributions: one, $\overline{\varepsilon_{VT}}$, takes into account the larger values (up to 20 times the mean value) measured inside the trailing vortex. The second, $\overline{\varepsilon_{OVT}}$, corresponds to the measured values outside the trailing vortex.

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