

## THE EFFECTS OF ECCENTRICITY AND DIAMETER OF HE 3 IMPELLER ON THE MOMENTUM TRANSFER PROCESS IN AN AGITATED VESSEL

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**Abstract.** In the paper, the results of the experimental studies concerning the effects of an eccentricity and a diameter of the HE 3 impeller on the momentum transfer process in an agitated vessel have been analyzed. Experiments were carried out in the un-baffled vessel of inner diameter  $T = 0.3$  m, using computer-aided electrochemical method. Distributions of the transport coefficient in the region of the vessel wall were numerically integrated. Averaged values of the shear rate, friction coefficient, shear stress and dynamic velocity as a function of the impeller eccentricity and diameter, as well as modified Reynolds number are correlated by means of suitable equations.

**Key words:** agitation, momentum transfer, eccentricity of the impeller, distribution of the transport coefficients

### 1. INTRODUCTION

Eccentric location of the impeller shaft affects the perturbation of the liquid flow symmetry in the agitated vessel. The effects of the impeller eccentricity on the hydrodynamic conditions in an agitated vessel have been recently studied by Hall et al. [1, 2], Montante et al. [3] and Karcz et al. [4 - 9]. Hall et al. [1, 2] analyzed the field of the flow velocity and mixing time using two-dimensional PIV and PLIF techniques. The investigations were carried out for a liquid and a gas – liquid system agitated using impellers located in a centric or eccentric position in the vessel. Experimentally obtained distributions of the velocity vectors were compared with the CFD results. Montante et al. [3] used PIV method to study hydrodynamics of the agitated vessel with a Rushton turbine located centrally or eccentrically. Mean flow velocities were compared with the numerical results. Karcz and Cudak [8, 9] investigated an effect of the impeller eccentricity on the mean and local values of the heat transfer coefficient. Experimental studies were carried out in a jacketed agitated vessel equipped with an axial-flow or radial-flow impeller. Mean values of this coefficient were measured using thermal method, whereas local values were determined by means of an electrochemical method.

The purpose of the experimental studies presented in the paper is to analyse the momentum transfer process in a region of the wall of an agitated vessel equipped with an eccentrically located HE 3 impeller with diameter  $D/T = 0.33$  or  $D/T = 0.5$ .

## 2. EXPERIMENTAL

The experiments were performed in an un-baffled agitated vessel of inner diameter  $T = 2R = 0.3$  m. Liquid height was equal to  $H = T$ . Down-pumping HE 3 impeller of diameter  $D = 0.33T$  (or  $D = 0.5T$ ) was located vertically at the distance  $h = 0.33T$  from the flat bottom of the vessel. Centric ( $e_R = 0$ ) and eccentric ( $e_R \in \langle 0.2; 0.8 \rangle$ ) positions of the HE 3 impeller were tested. Impeller eccentricity  $e$  was defined by the distance between a vessel and a shaft axes (Fig. 1a, b). Dimensionless impeller eccentricity was determined as follows  $e_R = 2e/(T - D)$ . In total, 112 electrochemical sensors identified by axial and angular co-ordinates  $z/H \in \langle 0.08; 0.92 \rangle$  and  $\varphi/2\pi \in \langle 0; 1 \rangle$ , were built on the vessel wall. Geometrical parameters of the agitated vessel and positions of the measuring cathodes on the cylindrical wall are shown in Fig. 1.

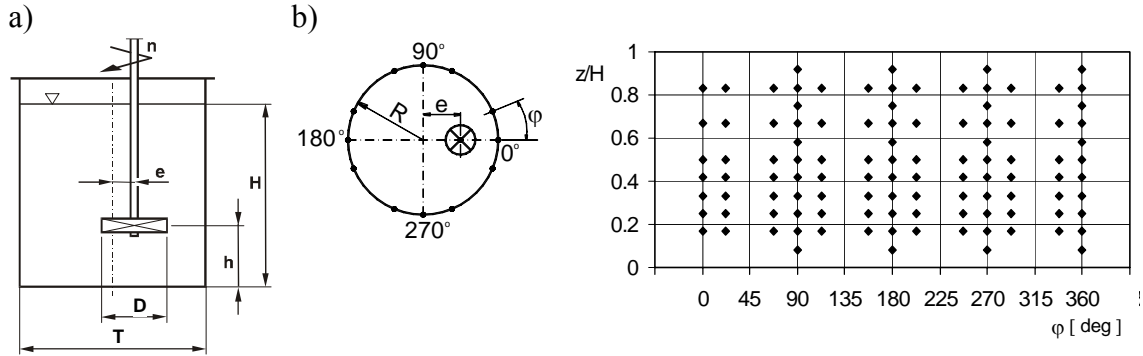


Fig. 1. Geometrical parameters of the agitated vessel and positions of the measuring cathodes on the cylindrical wall

Power consumption was measured using strain gauge method. Electrochemical measurements were carried for different agitator speeds  $n$ , within the turbulent regime of the Newtonian liquid flow ( $Re \in \langle 1.9 \times 10^4; 7.5 \times 10^4 \rangle$ ) in the agitated vessel. The aqueous solution of the electrolyte consisted of the  $0.01 \text{ kmol/m}^3$  potassium ferricyanide  $\text{K}_3\text{Fe}(\text{CN})_6$ ,  $0.05 \text{ kmol/m}^3$  potassium ferrocyanide  $\text{K}_4\text{Fe}(\text{CN})_6$  and sodium hydroxide  $\text{NaOH}$  of the concentration  $0.5 \text{ kmol/m}^3$ . Viscosity of the electrolyte solution was approximately equal to  $\eta = 1.24 \times 10^{-3}$  Pas. The process of the momentum transfer in the vicinity of the agitated vessel wall was studied experimentally, using computer aided electrochemical method, which is described in details elsewhere [4, 10]. In each measuring point (see Fig. 1b), the measuring digital voltage signal was sampled 200 times. Diffusion current  $I_d$  representative for a given measuring point was automatically calculated on the basis of the averaged value from the sampling. Size of the sampling was chosen experimentally. The tests showed that representative locally averaged values for the measuring point, obtained from the 200 and 500 samples, did not differ statistically. In the electrochemical method, local values of the: shear rate  $\gamma$ , shear stress  $\tau$ , dynamic velocity  $v^*$  and friction coefficient  $f$  proportional to the diffusion current  $I_d$ , are calculated as follows

- shear rate  $\gamma$  [11, 12]

$$\gamma = \left( \frac{I_d}{2.156 z_e F D_A^{\frac{2}{3}} C_{A0} R_c^{\frac{5}{3}}} \right)^3 \quad (1)$$

- shear stress  $\tau$

$$\tau = \eta \gamma \quad (2)$$

- dynamic velocity  $v^*$

$$v^* = \sqrt{\frac{\tau}{\rho}} \quad (3)$$

- friction coefficient  $f$  [13, 14]

$$f = \frac{\tau D^2}{\rho n^2 d^4} \quad (4)$$

### 3. RESULTS AND DISCUSSION

In total, the sets including of the 4000 local values of diffusion current  $I_d$  measured on the cylindrical wall of the agitated vessel wall, were obtained. In Figs. 2 – 10, the results of the experiments, are compared for centric and eccentric positions of the HE 3 impeller in the vessel.

The effects of the axial  $z/H$  and angular  $\varphi$  coordinates on the dimensionless shear rate  $\gamma/n$ , depending on the impeller eccentricity and Reynolds number (where  $Re = nd^2\rho/\eta$ ), are presented in Fig. 2 – 4. Fig. 2 shows the results  $\gamma/n = F(z/H)_{\varphi=\text{const}}$  for the agitated vessel equipped with centric HE 3 impeller of diameter  $D = 0.33T$  (Fig. 2a) or  $D = 0.5T$  (Fig. 2b). The greatest values of the  $\gamma/n$  correspond to the impeller region, i.e. dimensionless axial coordinate  $z/H \in <0.2; 0.4>$ . Shape of the function distribution depends on the impeller diameter. The values of the dimensionless shear rate increase with the increase of the  $Re$  number.

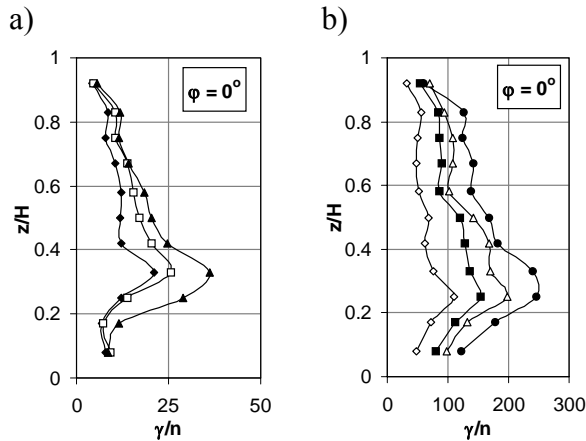


Fig. 2. The dependence  $\gamma/n = F(z/H)_{\varphi=\text{const}}$  for the agitated vessel with centrally located HE 3 impeller;  $e_R = 0$ ; a)  $D/T = 0.33$ ; b)  $D/T = 0.5$ ;  $\diamond$  -  $Re = 1.9 \times 10^4$ ;  $\blacklozenge$  -  $Re = 3.5 \times 10^4$ ;  $\blacksquare$  -  $Re = 3.7 \times 10^4$ ;  $\square$  -  $Re = 5.1 \times 10^4$ ;  $\triangle$  -  $Re = 5.6 \times 10^4$ ;  $\blacktriangle$  -  $Re = 6.6 \times 10^4$ ;  $\bullet$  -  $Re = 7.5 \times 10^4$

Axial distributions of the shear rate  $\gamma/n = F(z/H)_{\varphi=\text{const}}$ , obtained for the eccentric position of the HE 3 impeller of diameter  $D = 0.33T$  and  $D = 0.5T$ , are compared in Fig. 3 ( $D = 0.33T$ ) and Fig. 4 ( $D = 0.5T$ ) assuming a given value of angular coordinate  $\varphi = 0^\circ, 90^\circ, 180^\circ$  or  $270^\circ$ . When HE 3 impeller of diameter  $D = 0.33T$  was used then the highest values of the  $\gamma/n$  were obtained for the angular coordinate  $\varphi = 0^\circ$  (Fig. 3a). In the case of the HE 3 impeller of diameter  $D = 0.5T$ , the regions of the highest local values of the  $\gamma/n$  correspond to the angular coordinates equal to  $\varphi = 0^\circ$  and  $\varphi = 270^\circ$  (Figs. 4a and 4d). The distributions of the  $\gamma/n =$

$F(z/H)_{\varphi=\text{const}}$  presented in Figs. 3 and 4 show that extreme points are located on the wall on the level below the height  $h$  of the eccentric HE 3 impeller. Local maximum of the function  $\gamma/n = F(z/H)_{\varphi=\text{const}}$  corresponds to the dimensionless axial coordinate  $z/H = 0.25$ .

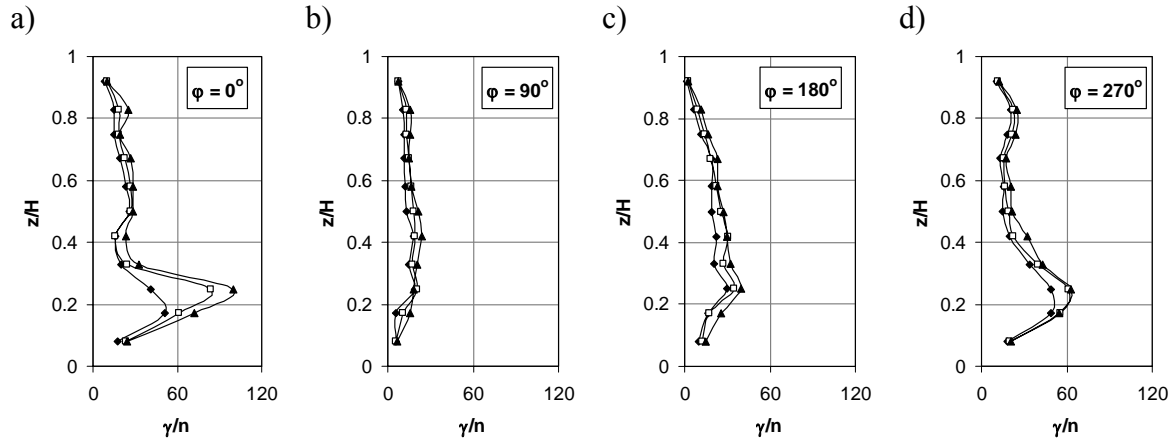


Fig. 3. The dependence  $\gamma/n = F(z/H)_{\varphi=\text{const}}$  for the agitated vessel with eccentrically located HE 3 impeller;  $e_R = 0.8$ ;  $D/T = 0.33$ ;  $\diamond$  -  $Re = 3.5 \times 10^4$ ;  $\square$  -  $Re = 5.1 \times 10^4$ ;  $\blacktriangle$  -  $Re = 6.6 \times 10^4$

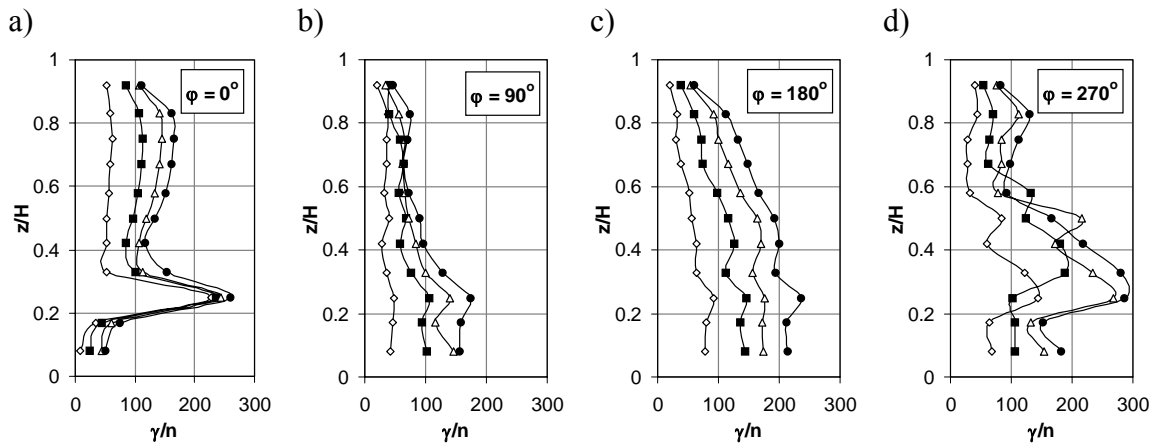


Fig. 4. The dependence  $\gamma/n = F(z/H)_{\varphi=\text{const}}$  for the agitated vessel with eccentrically located HE 3 impeller;  $e_R = 0.8$ ;  $D/T = 0.5$ ;  $\diamond$  -  $Re = 1.9 \times 10^4$ ;  $\blacksquare$  -  $Re = 3.7 \times 10^4$ ;  $\triangle$  -  $Re = 5.6 \times 10^4$ ;  $\bullet$  -  $Re = 7.5 \times 10^4$

The effects of the shaft eccentricity on the distribution of the friction coefficient  $f$  at vicinity of the vessel wall are presented in Fig. 5 ( $D/T = 0.33$ ) and Fig. 6 ( $D/T = 0.5$ ). The results are compared, assuming constant values of  $Re$  number and angular coordinate  $\varphi$ . On the basis of the results obtained it can be stated that, in general, values of the coefficient  $f$  increase with the increase of the impeller eccentricity, however, with the exception of the data ascribed to the angular coordinate  $\varphi = 90^\circ$  (Figs. 5b, 6b).

In this case, higher values of the friction coefficient  $f$  were obtained for the agitated vessel with centrally located HE 3 impeller, comparing with the results concerning the vessel equipped with the eccentric impeller ( $e_R \neq 0$ ). Moreover, the greatest values of the  $f$  correspond to the eccentricity  $e_R = 0.6$  of the smaller impeller ( $D/T = 0.33$ , Fig. 5c,d) and to the angular coordinates  $\varphi = 180^\circ$  and  $\varphi = 270^\circ$ .

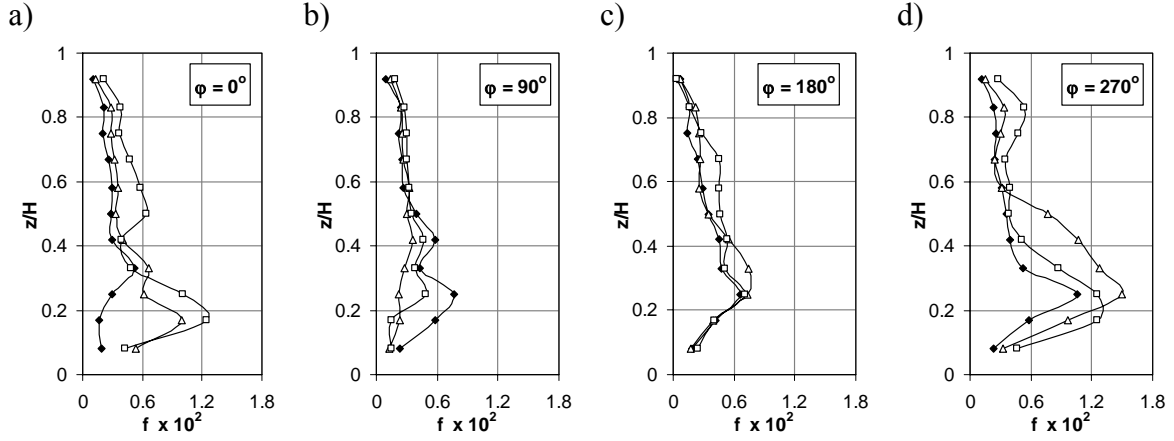


Fig. 5. The dependence  $f = F(z/H)_{\varphi=const}$  for the agitated vessel with centrally and eccentrically located HE 3 impeller;  $D/T = 0.33$ ;  $Re = 3.5 \times 10^4$ ;  $\blacklozenge$  -  $e_R = 0$ ;  $\triangle$  -  $e_R = 0.6$ ;  $\square$  -  $e_R = 0.8$

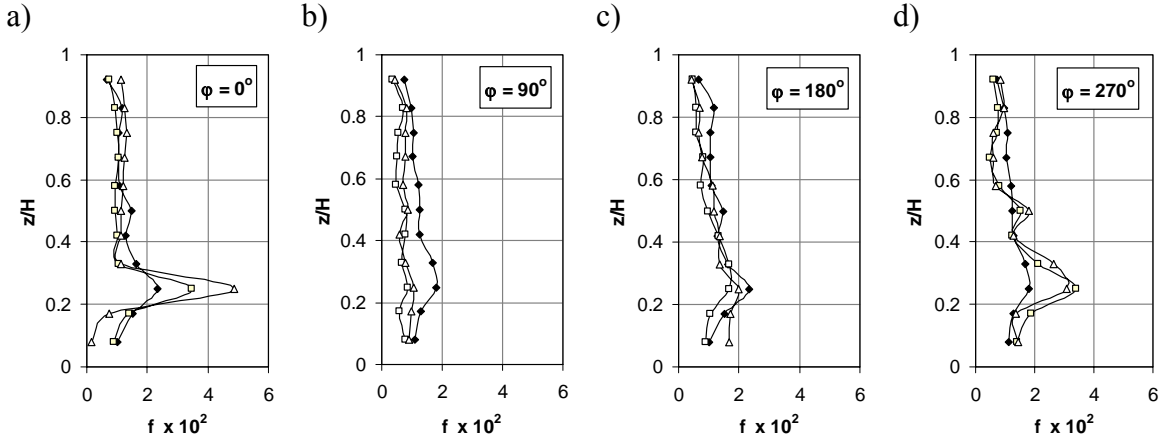


Fig. 6. The dependence  $f = F(z/H)_{\varphi=const}$  for the agitated vessel with centrally and eccentrically located HE 3 impeller;  $D/T = 0.5$ ;  $Re = 3.7 \times 10^4$ ;  $\blacklozenge$  -  $e_R = 0$ ;  $\square$  -  $e_R = 0.53$ ;  $\triangle$  -  $e_R = 0.8$

Radial profiles  $f = F(\varphi)$  for the agitated vessel equipped with centrally or eccentrically located both up-pumping propeller and down-pumping HE 3 impeller ( $D/T = 0.33$  and  $D/T = 0.5$ ) are compared in Figs. 7 – 8. In all the cases, the highest values of the  $f$  characterize the vessel with the HE 3 impeller of diameter  $D = 0.5T$ . The effect of the operating pumping mode of the impellers on the radial distributions of the  $f$  is found. For down-pumping HE 3 impeller ( $D = 0.33T$ ), local values of the friction coefficients are higher for the axial coordinate  $z/H = 0.25$  (Fig. 7a) than ones obtained for the  $z/H = 0.33$ . This relation is opposite for the up-pumping propeller. Moreover, the results show slight effect of the angular coordinate  $\varphi$  on the friction coefficient  $f$  for the centric position of the impeller (Fig. 7). This effect depends on the tangential component of the liquid velocity.

Axially averaged profiles of the functions  $\tau_m = F(\varphi)$  shown in Fig. 9, represent the data for the different type of the impellers and a constant value of the mean specific agitation energy  $\varepsilon_m = 0.05$  W/kg. Agitation energy  $\varepsilon_m$  was calculated from the independently measured power consumption using strain gauge method. The results obtained for the vessel with the centrally located impeller of this same diameter (Fig. 9a) show that values of the shear stress  $\tau_m$  are comparable and do not depend practically on the operating pumping mode of the impeller. However, this effect is revealed slightly when the impeller is situated eccentrically in the agitated vessel (Fig. 9b).

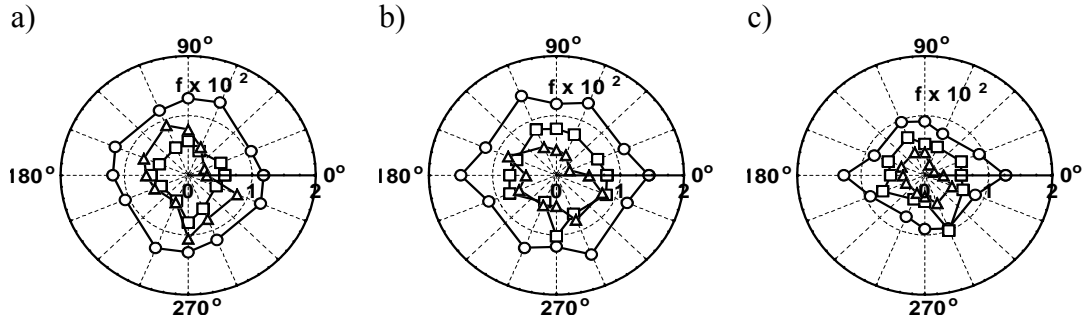


Fig. 7. The dependence  $f = F(\varphi)_{(z/H) = \text{const}}$  for the agitated vessel with centrally located impeller;  $e_R = 0$ ; a)  $z/H = 0.25$ ; b)  $z/H = 0.33$ ; c)  $z/H = 0.5$ ;  $\square$  - propeller;  $D/T = 0.33$ ;  $Re = 3.6 \times 10^4$ ;  $\circ$  - HE 3;  $D/T = 0.5$ ;  $Re = 3.7 \times 10^4$ ;  $\Delta$  - HE 3;  $D/T = 0.33$ ;  $Re = 3.5 \times 10^4$

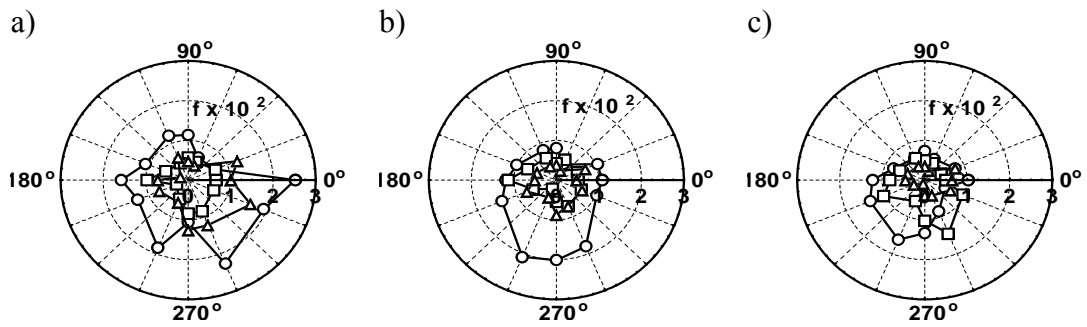


Fig. 8. The dependence  $f = F(\varphi)_{(z/H) = \text{const}}$  for the agitated vessel with eccentrically located impeller;  $e_R = 0.8$ ; a)  $z/H = 0.25$ ; b)  $z/H = 0.33$ ; c)  $z/H = 0.5$ ;  $\square$  - propeller;  $D/T = 0.33$ ;  $Re = 3.6 \times 10^4$ ;  $\circ$  - HE 3;  $D/T = 0.5$ ;  $Re = 3.7 \times 10^4$ ;  $\Delta$  - HE 3;  $D/T = 0.33$ ;  $Re = 3.5 \times 10^4$

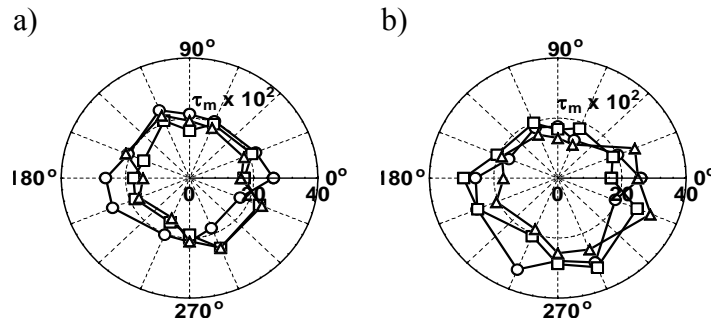


Fig. 9. The dependence  $\tau_m = F(\varphi)_{(z/H) = \text{const}}$  for the agitated vessel with centrally or eccentrically located impeller;  $\varepsilon_m = 0.05 \text{ W/kg}$ ; a)  $e_R = 0$ ; b)  $e_R = 0.8$ ;  $\square$  - propeller;  $D/T = 0.33$ ;  $Re = 5.8 \times 10^4$  ( $e_R = 0$  and  $e_R = 0.8$ );  $\circ$  - HE 3;  $D/T = 0.5$ ;  $Re = 3.7 \times 10^4$  ( $e_R = 0$  and  $e_R = 0.8$ );  $\Delta$  - HE 3;  $D/T = 0.33$ ;  $Re = 6.3 \times 10^4$  ( $e_R = 0$ ) and  $Re = 5.6 \times 10^4$  ( $e_R = 0.8$ )

Two-dimensional distributions of the dimensionless shear rate on the vessel wall,  $\gamma/n = F(z/H, \varphi/2\pi)$  for  $Re = \text{const}$ , obtained for the eccentric position ( $e_R = 0.8$ ) of the HE 3 impellers differing in the diameter are compared in Fig. 10. In both cases, the region of the greatest dimensionless shear rate  $\gamma/n$  is located on the level of the impeller height and within the range of the dimensionless angular coordinate  $\varphi/2\pi \in \langle 0.7; 0.9 \rangle$ .

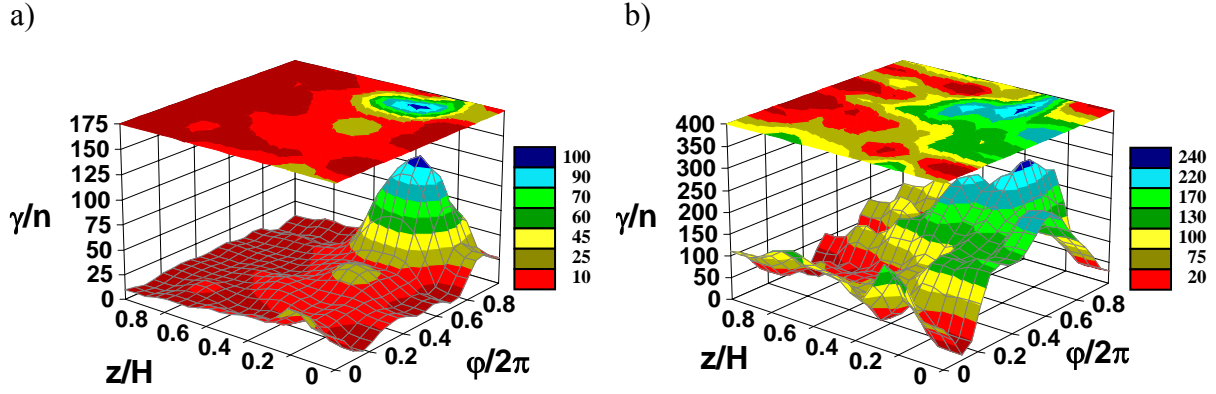


Fig. 10. The dependence  $\gamma/n = F(z/H, \phi/2\pi)$  for the agitated vessel with eccentrically located HE 3 impeller;  $e_R = 0.8$ ;  $Re = 5.6 \times 10^4$ ; a)  $D/T = 0.33$ ; b)  $D/T = 0.5$

Local values of the dimensionless shear rate  $\gamma/n$ , friction coefficient  $f$ , shear stress  $\tau$  and dynamic velocity  $v^*$  have been numerically integrated for the whole surface area of the cylindrical wall of the agitated vessel ( $0 < z/H < 1$  and  $0 < \phi/2\pi < 1$ ). The dependences of the averaged  $(\gamma/n)_m$ ,  $f_m$ ,  $\tau_m$ , and  $v_m^*$  on the eccentricity ratio  $e_R$ , diameter of the HE 3 impeller  $D$  and modified Reynolds number  $Re_{P,M}$  were approximated with mean relative error  $\pm 5\%$  using following five-parameter equations

$$\left(\frac{\gamma}{n}\right)_m = 0.037 Re_{P,M}^{0.20} (0.421e_R^2 - 0.343e_R + 1) \left(\frac{3D}{T}\right)^{2.89} \quad (5)$$

$$f_m = 0.088 Re_{P,M}^{-0.10} (0.684e_R^2 - 0.454e_R + 1) \left(\frac{3D}{T}\right)^{2.42} \quad (6)$$

$$\tau_m = 1.883 \cdot 10^{-8} Re_{P,M}^{0.50} (0.222e_R^2 - 0.257e_R + 1) \left(\frac{3D}{T}\right)^{-0.34} \quad (7)$$

$$v_m^* = 3.951 \cdot 10^{-6} Re_{P,M}^{0.25} (0.203e_R^2 - 0.216e_R + 1) \left(\frac{3D}{T}\right)^{-0.08} \quad (8)$$

Modified Reynolds number was defined as follows:  $Re_{P,M} = ((P/M)\rho^3 T^4 / \eta^3)$ , where  $P$  – power consumption measured using strain gauge method;  $M = V\rho$ . The Eqs. (5) - (8) describe the results of the measurements within the following range of the variables:  $e_R \in <0; 0.8>$ ,  $D/T \in <0.33; 0.5>$  and  $Re_{P,M} \in <1.78 \times 10^{13}; 2.21 \times 10^{15}>$ .

#### 4. CONCLUSIONS

On the basis of the experimental studies performed within the turbulent regime of the Newtonian liquid flow it can be stated that

1. Deformations of the axial and angular distributions of the momentum transfer coefficients on the cylindrical wall of the vessel increase with the increase of the impeller eccentricity.
2. Dimensionless shear rate and friction coefficient depend on the diameter of the HE 3 impeller. Values of those parameters increase with the increase of the impeller diameter.
3. Shear stress values  $\tau_m$  compared at assumed constant value of the mean specific energy  $\varepsilon_m$  slightly only depend on the impeller diameter and pumping mode of the impeller.

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## NOMENCLATURE

$C_{A0}$ – concentration of component A kmol/m <sup>3</sup>	$R_c$ – radius of cathode, m
$D_A$ – diffusivity m <sup>2</sup> /s	$T$ – inner diameter of the vessel, m
$D$ – diameter of the impeller, m	$v^*$ – dynamic velocity, m/s
$e$ – eccentricity of the impeller shaft, m	$z$ – axial coordinate, m
$e_R$ – dimensionless eccentricity ( $= 2e/(T-D)$ )	$z_e$ – number of electrons taking part in a reaction
$F$ – Faraday's constant, C/kmol	$\varepsilon$ – energy dissipated, W/kg
$f$ – friction coefficient	$\gamma$ – shear rate, 1/s
$H$ – liquid height in the vessel, m	$\eta$ – dynamic viscosity, Pas
$h$ – off – bottom clearance of the impeller, m	$\varphi$ – angular coordinate, deg
$I_d$ – diffusion current, A	$\rho$ – liquid density, kg/m <sup>3</sup>
$n$ – agitator speed, 1/s	$\tau$ – shear stress, N/m <sup>2</sup>
$R$ – radius of the agitated vessel ( $= D/2$ ), m	

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